

EXERCISE 3.1

Q.1 Express the following as a ratio $a : b$ and as a fraction in its simplest (lowest) form.

(i) Rs. 750 : Rs. 1250

$$= \frac{\text{Rs.}750}{\text{Rs.}1250} = \frac{75 \div 25}{125 \div 25} = \frac{3}{5} = 3 : 5$$

(ii) 450 cm : 3 m ($\because 1\text{m} = 100\text{cm}$)
 450 cm : 300 cm

$$= \frac{450\cancel{\text{cm}}}{300\cancel{\text{cm}}} = \frac{45}{30} = \frac{45 \div 15}{30 \div 15} = \frac{3}{2} = 3 : 2$$

(iii) 4kg : 2kg 750g ($\because 1\text{kg} = 1000\text{g}$)

$$= 4000\text{g} : (2000\text{g} + 750\text{g})$$

$$= 4000\text{g} : 2750\text{g}$$

$$= 4000\text{g} : 2750\text{g}$$

$$= \frac{4000\cancel{\text{g}}}{2750\cancel{\text{g}}} = \frac{400 \div 25}{275 \div 25} = \frac{16}{11} = 16 : 11$$

(iv) 27min. 30 Second , 1 h
 27min. 30 Second : 1 h

$$\because 1\text{h} = 60\text{min} = 60 \times 60\text{s} = 3600\text{s}$$

$$= (27 \times 60\text{ s} + 30\text{ s}) : 3600\text{ s}$$

$$= (1620\text{ s} + 30\text{ s}) : 3600\text{ s}$$

$$= 1650\text{ s} : 3600\text{ s}$$

$$= \frac{1650}{3600} = \frac{165}{360} \text{ (dividing by 15)}$$

$$= \frac{11}{24} = 11 : 24$$

(v) $75^\circ : 225^\circ$

$$= \frac{75 \times 1^\circ}{225 \times 1^\circ} = \frac{75 \div 25}{225 \div 25} = \frac{3}{9}$$

$$= \frac{1}{3} = 1 : 3$$

Q.2 In a class of 60 students, 25 students are girls and remaining students are boys. Compute the ratio of

- (i). Boys to total students
 (ii). Boys to girls

Solution:

Number of students = 60
 Number of girls = 25
 Number of boys = $60 - 25 = 35$

(i) Ratio of boys to the total students
 Number of boys : Total students
 $35 : 60$

$$= \frac{35}{60} = \frac{35 \div 5}{60 \div 5} = \frac{7}{12} = 7 : 12$$

(ii) Ratio of boys to the girls
 Number of boys : Number of girls.
 $35 : 25$

$$= \frac{35}{25} = \frac{35 \div 5}{25 \div 5} = \frac{7}{5} = 7 : 5$$

Q.3 If $3(4x-5y) = 2x-7y$, find the ratio $x : y$.

Solution: $3(4x-5y) = 2x-7y$

$$12x - 15y = 2x - 7y$$

$$12x - 2x = 15y - 7y$$

$$10x = 8y$$

$$\frac{x}{y} = \frac{8}{10}$$

$$\frac{x}{y} = \frac{8 \div 2}{10 \div 2}$$

$$\frac{x}{y} = \frac{4}{5}$$

$$x : y = 4 : 5$$

Q.4 Find the value of p , if the ratios $2p + 5 : 3p + 4$ and $3 : 4$ are equal.

Solution: 1st ratio = $2p + 5 : 3p + 4$

2nd ratio = $3 : 4$

According to given condition

$$2p + 5 : 3p + 4 = 3 : 4$$

$$\frac{2p + 5}{3p + 4} = \frac{3}{4}$$

$$4(2p + 5) = 3(3p + 4)$$

$$8p + 20 = 9p + 12$$

$$20 - 12 = 9p - 8p$$

$$8 = p$$

$$\Rightarrow \boxed{p = 8}$$

Q.5 If the ratios $3x + 1 : 6 + 4x$ and $2 : 5$ are equal. Find the value of x .

Solution: 1st Ratio = $3x + 1 : 6 + 4x$

2nd Ratio = $2 : 5$

According to given condition

$$3x + 1 : 6 + 4x = 2 : 5$$

$$\frac{3x+1}{6+4x} = \frac{2}{5}$$

$$5(3x+1) = 2(6+4x)$$

$$15x + 5 = 12 + 8x$$

$$15x - 8x = 12 - 5$$

$$7x = 7$$

$$x = \frac{7}{7} = 1$$

$$\boxed{x = 1}$$

Q.6 Two numbers are in the ratio $5 : 8$. If 9 is added to each number, we get a new ratio $8 : 11$. Find the numbers.

Solution: Ratio between two numbers = $5 : 8$

Number to be added = 9

New ratio = $8 : 11$

Let 1st Number = $5x$

2nd Number = $8x$

By condition:

$$5x + 9 : 8x + 9 = 8 : 11$$

$$\frac{5x+9}{8x+9} = \frac{8}{11}$$

$$11(5x+9) = 8(8x+9)$$

$$55x + 99 = 64x + 72$$

$$99 - 72 = 64x - 55x$$

$$27 = 9x$$

$$\frac{27}{9} = x$$

$$3 = x \Rightarrow x = 3$$

Thus 1st number = $5x = 5(3) = 15$

2nd number = $8x = 8(3) = 24$

Required numbers are 15 and 24.

Q.7 If 10 is added in each number of the ratio $4 : 13$, we get a new ratio $1 : 2$. What are the numbers?

Solution: Ratio of two numbers = $4 : 13$

Number to be added = 10

New ratio = $1 : 2$

Let 1st number = $4x$

2nd number = $13x$

By condition:

$$4x + 10 : 13x + 10 = 1 : 2$$

$$\frac{4x+10}{13x+10} = \frac{1}{2}$$

$$2(4x+10) = 1(13x+10)$$

$$8x + 20 = 13x + 10$$

$$20 - 10 = 13x - 8x$$

$$10 = 5x$$

$$\frac{10}{5} = x$$

$$2 = x \Rightarrow \boxed{x = 2}$$

Thus 1st number = $4x = 4(2) = 8$

2nd number = $13x = 13(2) = 26$

Required numbers are 8 and 26.

Q.8 Find the cost of 8 kg of mangoes, if 5 kg of mangoes cost Rs. 250.

Solution: Mangoes = 5kg

Price = Rs. 250

Mangoes = 8kg

Let Price = $x = ?$

We know that

$5\text{kg} : 8\text{kg} = \text{Rs. } 250 : \text{Rs. } x$

$$\frac{5}{8} = \frac{250}{x}$$

$$x \times 5 = 8 \times 250$$

$$x = \frac{8 \times 250}{5}$$

$$x = 8 \times 50$$

$$x = \text{Rs. } 400$$

Q.9 If $a : b = 7 : 6$, find the value of $3a + 5b : 7b - 5a$.

Solution: $a : b = 7 : 6$

$$\frac{a}{b} = \frac{7}{6}$$

$$3a + 5b : 7b - 5a = \frac{3a + 5b}{7b - 5a}$$

Dividing numerator and denominator by "b".

$$\frac{3a + 5b}{7b - 5a} = \frac{\frac{3a}{b} + 5}{7 - 5\frac{a}{b}}$$

$$= \frac{3\frac{a}{b} + 5}{7 - 5\frac{a}{b}}$$

Putting $\frac{a}{b} = \frac{7}{6}$, we get

$$\begin{aligned} &= \frac{3\left(\frac{7}{6}\right) + 5}{7 - 5\left(\frac{7}{6}\right)} \\ &= \frac{\frac{21}{6} + 5}{7 - \frac{35}{6}} = \frac{\frac{21+30}{6}}{\frac{42-35}{6}} = \frac{51}{7} \end{aligned}$$

$$3a + 5b : 7b - 5a = 51 : 7$$

Q.10 Complete the following:

(i) If $\frac{24}{7} = \frac{6}{x}$, then $4x = \dots\dots\dots$

(ii) If $\frac{5a}{3x} = \frac{15b}{y}$, then $ay = \dots\dots\dots$

(iii) If $\frac{9pq}{2lm} = \frac{18p}{5m}$, then $5q = \dots\dots\dots$

Solution:

(i) $\frac{24}{7} = \frac{6}{x}$
 $24x = 7 \times 6$
 $6 \times 4x = 7 \times 6$

$$4x = \frac{7 \times 6}{6}$$

$$\boxed{4x = 7}$$

(ii) $\frac{5a}{3x} = \frac{15b}{y}$

$$5ay = 15b(3x)$$

$$5ay = 45bx$$

$$ay = \frac{45bx}{5}$$

$$\boxed{ay = 9bx}$$

(iii) $\frac{9pq}{2lm} = \frac{18p}{5m}$

$$5m(9pq) = 18p(2lm)$$

$$5q = \frac{36plm}{9pm}$$

$$5q = \frac{\cancel{36}^4 \cancel{p} \cancel{l} \cancel{m}}{\cancel{9} \cancel{p} \cancel{m}}$$

$$\boxed{5q = 4l}$$

Q.11 Find x in the following proportions.

(i) $3x - 2 : 4 :: 2x + 3 : 7$

(ii) $\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$

(iii) $\frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$

(iv) $p^2 + pq + q^2 : x :: \frac{p^3 - q^3}{p+q} : (p-q)^2$

(v) $8 - x : 11 - x :: 16 - x : 25 - x$

Solution:

(i) $3x - 2 : 4 :: 2x + 3 : 7$

Product of Extremes = Product of Means

$$7(3x - 2) = 4(2x + 3)$$

$$21x - 14 = 8x + 12$$

$$21x - 8x = 12 + 14$$

$$13x = 26$$

$$x = \frac{26}{13}$$

$$\boxed{x = 2}$$

$$(ii) \quad \frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$$

Product of Extremes = Product of Means

$$\left(\frac{3x-1}{7}\right)\left(\frac{7}{5}\right) = \left(\frac{3}{5}\right)\left(\frac{2x}{3}\right)$$

$$\frac{3x-1}{5} = \frac{2x}{5}$$

$$3x-1 = \frac{2x}{5} \times 5$$

$$3x-1 = 2x$$

$$3x-2x = 1$$

$$\boxed{x = 1}$$

$$(iii) \quad \frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$$

Product of Extremes = Product of Means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right) = \left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$

$$\frac{4x-12}{2x+8} = \frac{5}{3}$$

$$3(4x-12) = 5(2x+8)$$

$$12x-36 = 10x+40$$

$$12x-10x = 40+36$$

$$2x = 76$$

$$x = \frac{76}{2}$$

$$\boxed{x = 38}$$

$$(iv) \quad p^2 + pq + q^2 : x :: \frac{p^3 - q^3}{p+q} : (p-q)^2$$

Product of Means = Product of Extremes

$$(x) \cdot \frac{(p^3 - q^3)}{(p+q)} = (p^2 + pq + q^2)(p-q)^2$$

$$x = \frac{(p^2 + pq + q^2)(p-q)^2(p+q)}{p^3 - q^3}$$

$$x = \frac{\cancel{(p^2 + pq + q^2)} \cancel{(p-q)} (p-q)(p+q)}{\cancel{(p-q)} \cancel{(p^2 + pq + q^2)}}$$

$$x = (p-q)(p+q)$$

$$\boxed{x = p^2 - q^2}$$

$$(v) \quad 8-x : 11-x :: 16-x : 25-x$$

Product of Extremes = Product of Means

$$(8-x)(25-x) = (11-x)(16-x)$$

$$200 - 8x - 25x + x^2 = 176 - 11x - 16x + x^2$$

$$200 - 33x + x^2 = 176 - 27x + x^2$$

$$200 - 176 = 33x - \cancel{x^2} - 27x + \cancel{x^2}$$

$$24 = 6x$$

$$\frac{24}{6} = x$$

$$4 = x$$

$$\Rightarrow \boxed{x = 4}$$