EXERCISE 3.2

Q.1 If y varies directly as x, and $y = 8$ when $x = 2$, find

(i) y in terms of x

(ii) y when $x = 5$

(iii) x when $y = 28$

Solution: $\boxed{y \propto x}$ $\boxed{\begin{array}{c} y = 8 \\ x = 2 \end{array}}$ $\boxed{\begin{array}{c} y = ? \\ x = 5 \end{array}}$ $\boxed{\begin{array}{c} x = ? \\ y = 28 \end{array}}$

(i) $y \propto x$

        $y = kx$ ……………..(i)

To find k,

Put $y = 8$ and $x = 2$ in equation (i)

        $8 = k\,(2)$

        $\dfrac{8}{2} = k$

        $4 = k \Rightarrow$ $\boxed{k = 4}$

(ii) To find y,

Put $k = 4$ in equation (i)

        $y = kx$

        $y = 4x$ …………..(ii)

Now Put $x = 5$ in equation (ii)

        $y = 4x$

        $y = 4(5)$

        $\boxed{y = 20}$

(iii) To find x,

Put $y = 28$ in equation (ii)

        $y = 4x$

        $28 = 4(x)$

        $28 = 4(x)$

        $7 = x \Rightarrow$ $\boxed{x = 7}$

Q.2 If $y \propto x$, and $y = 7$ when $x = 3$ find

(i) y in terms of x

(ii) x when $y = 35$ and y when $x = 18$

Solution:

$\boxed{y \propto x}$ $\boxed{\begin{array}{c} y = 7 \\ x = 3 \end{array}}$ $\boxed{\begin{array}{c} x = ? \\ y = 35 \end{array}}$ $\boxed{\begin{array}{c} x = 18 \\ y = ? \end{array}}$

(i) $y \propto x$

        $y = kx$ ………………(i)

To find k,

Put $y = 7$ and $x = 3$ in equation (i)

        $7 = k\,(3)$

        $\dfrac{7}{3} = k \Rightarrow$ $\boxed{k = \dfrac{7}{3}}$

Put it in equation (i)

        $y = kx$

        $y = \dfrac{7}{3}x$ …………..(ii)

To find x,

Put $y = 35$ in equation (ii)

        $y = \dfrac{7}{3}x$

        $35 = \dfrac{7}{3}x$

        $\dfrac{3 \times 35}{7} = x$

        $3 \times 5 = x$

        $15 = x \Rightarrow$ $\boxed{x = 15}$

To find y,

Put $k = \dfrac{7}{3}$ and $x = 18$ in equation (i)

        $y = kx$

        $y = \dfrac{7}{\cancel{3}}\left(\overset{6}{\cancel{18}}\right)$

        $y = 7(6)$

        $\boxed{y = 42}$

Q.3 If $R \propto T$ and $R = 5$ when $T = 8$, find the equation connecting R and T. Also find R when $T = 64$ and T when $R = 20$.

Solution: $\boxed{R \propto T}$ $\boxed{\begin{array}{c} R = 5 \\ T = 8 \end{array}}$ $\boxed{\begin{array}{c} R = ? \\ T = 64 \end{array}}$ $\boxed{\begin{array}{c} R = 20 \\ T = ? \end{array}}$

        $R \propto T$

        $R = KT$ ……………..(i)

To find k,

Put $R = 5$ and $T = 8$ in equation (i)

        $5 = K\,(8)$

        $\dfrac{5}{8} = k \Rightarrow$ $\boxed{k = \dfrac{5}{8}}$

Put $k = \dfrac{5}{8}$ in equation (i)

R=kT

R=$\dfrac{5}{8}$T ……………………(ii)

To find *R*,

Put $T = 64$ in equation (ii)

$$R = \dfrac{5}{8}T$$

$$R = \dfrac{5}{\cancel{8}}\left(\overset{8}{\cancel{64}}\right)$$

$$R = 5(8)$$

$$\boxed{R = 40}$$

To find *T*,

Put $K = \dfrac{5}{8}$ and $R = 20$ in equation (i)

$$R = KT$$

$$20 = \dfrac{5}{8}(T)$$

$$T = \dfrac{\overset{4}{\cancel{20}} \times 8}{\cancel{5}}$$

$$4 \times 8 = T$$

$$32 = T \qquad \Rightarrow \qquad \boxed{T = 32}$$

Q.4 If $R \propto T^2$ and $R = 8$ when T = 3, find R when T = 6.

Solution: $\boxed{R \propto T^2}$ $\boxed{\begin{array}{l} R = 8 \\ T = 3 \end{array}}$ $\boxed{\begin{array}{l} R = ? \\ T = 6 \end{array}}$

$$R \propto T^2$$
$$R = kT^2 \quad \text{…………………(i)}$$

To find k,

Put $R = 8$ and $T = 3$ in equation (i)

$$8 = k(3)^2$$

$$8 = k(9)$$

$$\dfrac{8}{9} = k \quad \Rightarrow \quad \boxed{k = \dfrac{8}{9}}$$

To find R,

Put $k = \dfrac{8}{9}$ and $T = 6$ in equation (i)

$$R = kT^2$$

$$R = \dfrac{8}{9}(6)^2$$

$$R = \dfrac{8\left(\overset{4}{\cancel{36}}\right)}{\cancel{9}}$$

$$R = 8(4) \qquad \Rightarrow \qquad \boxed{R = 32}$$

Q.5 $V \propto R^3$ and $V = 5$ when $R = 3$, find R, when V = 625.

Solution: $\boxed{V \propto R^3}$ $\boxed{\begin{array}{l} V = 5 \\ R = 3 \end{array}}$ $\boxed{\begin{array}{l} R = ? \\ V = 625 \end{array}}$

$$V \propto R^3$$
$$V = kR^3 \quad \text{……………..(i)}$$

To find k,

Put V = 5 and R = 3 in equation (i)

$$5 = k(3^3)$$

$$5 = k(27)$$

$$\dfrac{5}{27} = k \Rightarrow k = \dfrac{5}{27}$$

Put $k = \dfrac{5}{27}$ in equation (i)

$$V = kR^3$$

$$V = \dfrac{5}{27}R^3 \quad \text{………………..(ii)}$$

To find R,

Now put $V = 625$ in equation (ii)

$$(625) = \dfrac{5}{27}R^3$$

$$\dfrac{27 \times \overset{125}{\cancel{625}}}{\cancel{5}} = R^3$$

$$27 \times 125 = R^3$$

$$R^3 = 3^3 \times 5^3$$

$$R^3 = (3 \times 5)^3$$

$$R^3 = (15)^3$$

Taking cube root of both sides

$$\sqrt[3]{R^3} = \sqrt[3]{15^3}$$

$$R^{\overset{1}{3 \times \frac{1}{3}}} = 15^{\overset{1}{3 \times \frac{1}{3}}}$$

$$\boxed{R = 15}$$

Q.6 If w varies directly as u^3 and w = 81 when u = 3. Find w, when u = 5.

Solution: $\boxed{w \propto u^3}$ $\boxed{\begin{array}{c} w = 81 \\ u = 3 \end{array}}$ $\boxed{\begin{array}{c} w = ? \\ u = 5 \end{array}}$

$$w \propto u^3$$
$$w = ku^3 \quad \text{............(i)}$$

To find k,
Put $w = 81$ and u = 3 in equation (i)

$$81 = k(3)^3$$
$$81 = k(27)$$
$$\frac{81}{27} = k$$
$$3 = k \implies \boxed{k = 3}$$

Put $k = 3$ in equation (i)
$$w = ku^3$$
$$w = 3u^3 \quad \text{..................(ii)}$$

To find w,
Now put u = 5 in equation (ii)

$$w = 3(5)^3$$
$$w = 3(125)$$
$$\boxed{w = 375}$$

Q.7 If y varies inversely as x and y = 7 when x = 2, find y, when x = 126.

Solution: $\boxed{y \propto \dfrac{1}{x}}$ $\boxed{\begin{array}{c} y = 7 \\ x = 2 \end{array}}$ $\boxed{\begin{array}{c} y = ? \\ x = 126 \end{array}}$

$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x} \quad \text{..................(i)}$$

To find k,
Put y = 7 and x = 2

$$7 = \frac{k}{2}$$
$$7 \times 2 = k$$
$$14 = k \implies \boxed{k = 14}$$

Put $k = 14$ in equation (i)
$$y = \frac{k}{x} = \frac{14}{x}$$
$$\implies y = \frac{14}{x} \quad \text{..........(ii)}$$

To find y,
Now put x = 126 in equation (ii)

$$y = \frac{14}{x} = \frac{14}{126} = \frac{\cancel{14}^{\,1}}{\cancel{126}^{\,9}}$$

$$\boxed{y = \frac{1}{9}}$$

Q.8 If $y \propto \dfrac{1}{x}$ and y = 4 when x = 3, find x when y = 24.

Solution: $\boxed{y \propto \dfrac{1}{x}}$ $\boxed{\begin{array}{c} y = 4 \\ x = 3 \end{array}}$ $\boxed{\begin{array}{c} x = ? \\ y = 24 \end{array}}$

$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x} \quad \text{.............. (i)}$$

To find k,
Put y = 4 and x = 3 in equation (i)

$$4 = \frac{k}{3}$$
$$4 \times 3 = k$$
$$12 = k \implies \boxed{k = 12}$$

Put k = 12 in equation (i)
$$y = \frac{k}{x}$$
$$y = \frac{12}{x} \quad \text{..................(ii)}$$

To find x,
Now put y = 24 in equation (ii)

$$24 = \frac{12}{x}$$
$$24x = 12$$
$$x = \frac{12}{24} = \frac{\cancel{12}^{\,1}}{\cancel{24}^{\,2}}$$

$$\boxed{x = \frac{1}{2}}$$

Q.9 If $w \propto \dfrac{1}{z}$ and $w = 5$ when $z = 7$, find w when $z = \dfrac{175}{4}$

Solution: $\boxed{w \propto \dfrac{1}{z}}$ $\boxed{\begin{array}{l} w = 5 \\ z = 7 \end{array}}$ $\boxed{\begin{array}{l} w = ? \\ z = \dfrac{175}{4} \end{array}}$

$$w \propto \frac{1}{z} \qquad \text{(given)}$$

$$w = \frac{k}{z} \quad \text{................ (i)}$$

To find k,
Put $w = 5$ and $z = 7$ in equation (i)

$$5 = \frac{k}{7}$$
$$5 \times 7 = k$$
$$35 = k \qquad \Rightarrow \qquad \boxed{k = 35}$$

Put $K = 35$ in equation (i)

$$w = \frac{k}{z} = \frac{35}{z} \Rightarrow w = \frac{35}{z} \quad \text{..........(ii)}$$

To find w,

Now put $z = \dfrac{175}{4}$ in equation (ii)

$$w = \frac{35}{z} = 35 \div z$$

$$w = 35 \div \frac{175}{4}$$

$$w = 35 \times \frac{4}{175}$$

$$w = \frac{\cancel{140}}{\cancel{175}} \, \cancel{35}^{1} \times \frac{4}{\cancel{175}5}$$

$$\boxed{w = \frac{4}{5}}$$

Q.10 $A \propto \dfrac{1}{r^2}$ and $A = 2$ when $r = 3$, find r when $A = 72$.

Solution: $\boxed{A \propto \dfrac{1}{r^2}}$ $\boxed{\begin{array}{l} A = 2 \\ r = 3 \end{array}}$ $\boxed{\begin{array}{l} A = 72 \\ r = ? \end{array}}$

$$A \propto \frac{1}{r^2}$$

$$A = \frac{k}{r^2} \quad \text{.................. (i)}$$

To find k,
Put $A = 2$ and $r = 3$ in equation (i)

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$2 \times 9 = k$$

$$18 = k \quad \Rightarrow \quad \boxed{k = 18}$$

Put $k = 18$ in equation (i)

$$A = \frac{k}{r^2}$$

$$A = \frac{18}{r^2} \quad \text{..................(ii)}$$

To find r,
Now put $A = 72$ in equation (ii)

$$72 = \frac{18}{r^2}$$

$$72\,r^2 = 18$$

$$r^2 = \frac{18}{72}$$

$$r^2 = \frac{1}{4}$$

Taking square root of both sides

$$\sqrt{r^2} = \pm\sqrt{\frac{1}{4}}$$

$$\boxed{r = \pm\frac{1}{2}}$$

Q.11 $a \propto \dfrac{1}{b^2}$ and $a = 3$ when $b = 4$, find a, when $b = 8$.

Solution: $\boxed{a \propto \dfrac{1}{b^2}}$ $\boxed{\begin{array}{l} a = 3 \\ b = 4 \end{array}}$ $\boxed{\begin{array}{l} a = ? \\ b = 8 \end{array}}$

$$a \propto \frac{1}{b^2}$$

$$a = \frac{k}{b^2} \quad \text{..................(i)}$$

To find k,
Put $a = 3$ and $b = 4$ in equation (i)

$$3 = \frac{k}{(4)^2}$$

$$3 = \frac{k}{16}$$

$$3 \times 16\ k$$

$$48 = k \Rightarrow \boxed{k = 48}$$

Put K = 48 in equation ....(i)

$$a = \frac{k}{b^2} \Rightarrow a = \frac{48}{b^2}\ ....(ii)$$

To find a,

Now, put b = 8 in equation .....(ii)

$$a = \frac{48}{b^2} = \frac{48}{(8)^2} = \frac{48}{64}$$

$$\boxed{a = \frac{3}{4}}$$

Q.12 $V \propto \dfrac{1}{r^3}$ and **V = 5 when r = 3, find V**
when r = 6 and r when V = 320.
Solution:

$V \propto \dfrac{1}{r^3}$	$\begin{array}{l}V=5\\r=3\end{array}$	$\begin{array}{l}V=?\\r=6\end{array}$	$\begin{array}{l}v=320\\r=?\end{array}$

$$V \propto \frac{1}{r^3}$$

$$V = \frac{k}{r^3} \ ....(i)$$

To find k,

Put V = 5 and r = 3 in equation (i)

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$5 \times 27 = k \quad \Rightarrow \quad \boxed{k = 135}$$

Put k = 135 in equation .....(i)

$$V = \frac{135}{r^3} \ ....(ii)$$

To find V, Put r = 6 in equation (ii)

$$V = \frac{135}{(6)^3} = \frac{135}{216}$$

$$\boxed{V = \frac{5}{8}} (Dividing\ by\ 27)$$

To find r, put v = 320 in equation (ii)

$$V = \frac{135}{r^3}$$

$$320 = \frac{135}{r^3}$$

$$r^3 = \frac{135}{320} = \frac{27}{64}$$

$$r^3 = \frac{3^3}{4^3}$$

$$r^3 = \left(\frac{3}{4}\right)^3 \quad \text{taking cube root}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\left(\frac{3}{4}\right)^3}$$

$$\boxed{r = \frac{3}{4}}$$

Q.13 $m \propto \dfrac{1}{n^3}$ and **m = 2 when n = 4 find m**
when n = 6 and n when m = 432.
Solution:

$m \propto \dfrac{1}{n^3}$	$\begin{array}{l}m=2\\n=4\end{array}$	$\begin{array}{l}m=?\\n=6\end{array}$	$\begin{array}{l}n=?\\m=432\end{array}$

$$m \propto \frac{1}{n^3}$$

$$m = \frac{k}{n^3} \ ....(i)$$

To find k,

Put m = 2 and n = 4 in equation (i)

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{K}{64}$$

$$2 \times 64 = k$$

$$128 = k \quad \Rightarrow \quad \boxed{k = 128}$$

Put k = 128 in equation ....(i)

$$m = \frac{128}{n^3} \ ....(ii)$$

To find m,

put n = 6 in equation (ii)

$$m = \frac{128}{(6)^3} = \frac{128}{216} = \frac{32}{54} = \frac{16}{27}$$

$$\boxed{m = \frac{16}{27}}$$

To find n,

Put m = 432 in equation (ii)

$$m = \frac{128}{n^3}$$

$$432 = \frac{128}{n^3}$$

$$n^3 = \frac{128}{432} \quad (dividing \ by \ 16)$$

$$n^3 = \frac{8}{27}$$

$$n^3 = \frac{2^2}{3^3}$$

$$n^3 = \left(\frac{2}{3}\right)^3$$

Taking cube of root

$$\sqrt[3]{n^3} = \sqrt[3]{\left(\frac{2}{3}\right)^3}$$

$$\boxed{n = \frac{2}{3}}$$