EXERCISE 3.3

Q1: Find a third proportional

(i) 6, 12

Solution:

Let third proportional is x then 6,12, x

By proportion.

$6 : 12 :: 12 : x$

Product of Extremes = Product of Means

$6(x) = 12 \times 12$

$6(x) = 144$

$x = \frac{144}{6} = 24$

$\boxed{x = 24}$

(ii) $a^3, 3a^2$

Solution:

Let 3rd proportional is x then $a^3, 3a^2, x$

By proportion

$a^3 : 3a^2 :: 3a^2 : x$

Product of Extremes = Product of Means

$xa^3 = (3a^2)(3a^2)$

$xa^3 = 9a^4$

$x = \frac{9a^4}{a^3}$

$x = 9a^{4-3}$

$\boxed{x = 9a}$

(iii) $a^2 - b^2, \ a - b$

Solution:

Let 3rd proportional is x then $a^2 - b^2, a - b, x$

By proportion

$a^2 - b^2 : a - b :: a - b : x$

Product of Extremes = Product of Means

$x(a^2 - b^2) = (a-b)(a-b)$

$x = \frac{(a-b)(a-b)}{a^2 - b^2}$

$x = \frac{(a-b)\cancel{(a-b)}}{(a+b)\cancel{(a-b)}}$

$\boxed{x = \frac{a-b}{a+b}}$

(iv) $(x-y)^2, \ x^3 - y^3$

Solution:

Let 3rd proportional is "a" then $(x-y)^2, \ x^3 - y^3, a$

By proportion:

$(x-y)^2 : x^3 - y^3 :: x^3 - y^3 : a$

Product of Extremes = Product of Means

$a(x-y)^2 = (x^3 - y^3)(x^3 - y^3)$

$a = \frac{(x^3 - y^3)^2}{(x-y)^2}$

$a = \frac{[(x-y)(x^2 + xy + y^2)]^2}{(x-y)^2}$

$a = \frac{\cancel{(x-y)^2}(x^2 + xy + y^2)^2}{\cancel{(x-y)^2}}$

$\boxed{a = (x^2 + xy + y^2)^2}$

(v) $(x+y)^2, \ x^2 - xy - 2y^2$

Solution:

Let 3rd proportional is "a"

Then $(x+y)^2, \ x^2 - xy - 2y^2, a$

By Proportion:

$(x+y)^2 : x^2 - xy - 2y^2 :: x^2 - xy - 2y^2 : a$

Product of Extremes = Product of Means:

$a(x+y)^2 = (x^2 - xy - 2y^2)(x^2 - xy - 2y^2)$

$a = \frac{(x^2 - xy - 2y^2)^2}{(x+y)^2}$

$a = \frac{(x^2 - xy - y^2 - y^2)^2}{(x+y)^2}$

$a = \frac{(x^2 - y^2 - xy - y^2)^2}{(x+y)^2}$

$a = \frac{[(x+y)(x-y) - y(x+y)]^2}{(x+y)^2}$

$a = \frac{[(x+y)(x-y-y)]^2}{(x+y)^2}$

$= \frac{\cancel{(x+y)^2}(x-2y)^2}{\cancel{(x+y)^2}}$

$\boxed{a = (x - 2y)^2}$

(vi) $\frac{p^2-q^2}{p^3+q^3}, \frac{p-q}{p^2-pq+q^2}$

Solution:

Let 3^{rd} proportional is x

Then $\frac{p^2-q^2}{p^3+q^3}$, $\frac{p-q}{p^2-pq+q^2}$, x

By proportion.

$$\frac{p^2-q^2}{p^3+q^3} : \frac{p-q}{p^2-pq+q^2} :: \frac{p-q}{p^2-pq+q^2} : x$$

Product of Extremes = Product of Means.

$$x.\ \frac{p^2-q^2}{p^3+q^3} = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2}$$

$$x.\ \frac{p^2-q^2}{p^3+q^3} = \left[\frac{(p-q)}{(p^2-pq+q^2)}\right]^2$$

$$x = \frac{(p-q)^2}{(p^2-pq+q^2)^2} \times \frac{p^3+q^3}{(p^2-q^2)}$$

$$x = \frac{\cancel{(p-q)}(p-q)}{(p^2-pq+q^2)\cancel{^2}} \times \frac{\cancel{(p+q)}\ \cancel{(p^2-pq+q^2)}}{\cancel{(p+q)}\ \cancel{(p-q)}}$$

$$\boxed{x = \frac{p-q}{p^2-pq+q^2}}$$

Q.2 Find a fourth proportional

(i) **5, 8, 15**

Solution:

Let 4^{th} proportional is x then 5, 8, 15, x

By proportion

5 : 8 : : 15 : x

Product of Extremes = Product of Means

$$5(x) = 8\,(15)$$

$$x = \frac{8(\cancel{15}\,^3)}{\cancel{5}\,1}$$

$$x = 8(3)$$

$$\boxed{x = 24}$$

(ii) **$4x^4$, $2x^3$, $18x^5$**

Solution:

Let 4^{th} proportional is "a" then $4x^4$, $2x^3$, $18x^5$, a

By proportion

$$4x^4 : 2x^3 :: 18x^5 : a$$

Product of Extreme = Product of Means.

$$a(4x^4) = 2x^3(18x^5)$$

$$a = \frac{36x^8}{4x^4}$$

$$a = 9x^{8-4}$$

$$\boxed{a = 9x^4}$$

(iii) **$15a^5b^6$, $10a^2b^5$, $21a^3b^3$**

Solution:

Let 4^{th} proportional is x

then $15a^5b^6$, $10a^2b^5$, $21a^3b^3$, x

By proportion

$$15a^5b^6 : 10a^2b^5 :: 21a^3b^3 : x$$

Product of Extremes = Product of Means

$$x\left(15a^5b^6\right) = \left(10a^2b^5\right)\left(21a^3b^3\right)$$

$$x = \frac{14\,\cancel{210}\ \cancel{a^5}\,b^8}{\cancel{15}\ \cancel{a^5}\,b^6}$$

$$x = 14b^{8-6}$$

$$\boxed{x = 14b^2}$$

(iv) **$x^2-11x+24$; $(x-3)$, $(5x^4-40x^3)$**

Solution:

Let 4^{th} proportional is "a"

$x^2-11x+24$, $(x-3)$, $(5x^4-40x^3)$, a

By proportion

$$x^2-11x+24 : (x-3) :: 5x^4-40x^3 : a$$

Product of Extremes = Product of Means

$$a\,(x^2-11x+24) = (x-3)\,(5x^4-40x^3)$$

$$a = \frac{(x-3)(5x^4-40x^3)}{x^2-11x+24}$$

$$a = \frac{(x-3).5x^3(x-8)}{x^2-3x-8x+24}$$

$$a = \frac{5x^3(x-3)(x-8)}{x(x-3)-8(x-3)}$$

$$a = \frac{5x^3\cancel{(x-3)}\ \cancel{(x-8)}}{\cancel{(x-3)}\ \cancel{(x-8)}}$$

$$\boxed{a = 5x^3}$$

(v) $p^3+q^3, p^2-q^2, p^2-pq+q^2$

Solution:

Let 4^{th} proportional is x

$p^3+q^3, p^2-q^2, p^2-pq+q^2, x$

By proportion

$p^3+q^3, p^2-q^2, p^2-pq+q^2 : x$

Product of Extremes = Product of Means.

$$x(p^3+q^3) = (p^2-q^2)(p^2-pq+q^2)$$

$$x = \frac{(p^2-q^2)(p^2-pq+q^2)}{p^3+q^3}$$

$$x = \frac{\cancel{(p+q)}(p-q)\cancel{(p^2-pq+q^2)}}{\cancel{(p+q)}\cancel{(p^2-pq+q^2)}}$$

$$\boxed{x = (p-q)}$$

(vi) $(p^2-q^2)(p^2+pq+q^2), p^3+q^3, p^3-q^3$

Solution:

Let 4^{th} proportional is x.

Then $(p^2-q^2)(p^2+pq+q^2), p^3+q^3, p^3-q^3, x$

By proportion:

$(p^2-q^2)(p^2+pq+q^2) : p^3+q^3 : : p^3-q^3 : x$

Product of Extremes = Product of Means:

$$x\ (p^2-q^2)(p^2+pq+q^2) = (p^3+q^3)(p^3-q^3)$$

$$x = \frac{(p^3+q^3)(p^3-q^3)}{(p^2-q^2)(p^2+pq+q^2)}$$

$$x = \frac{\cancel{(p+q)}(p^2-pq+q^2)\cancel{(p-q)}\cancel{(p^2+pq+q^2)}}{\cancel{(p+q)}\cancel{(p-q)}\cancel{(p^2+pq+q^2)}}$$

$$\boxed{x = (p^2-pq+q^2)}$$

Q.3: Find mean proportional:

(i) 20, 45

Solution:

Let mean proportional is m

then 20, m, 45

By proportion

20 : m : : m : 45

Product of Means = Product of Extremes

$$m.m = 20 \times 45$$

$$m^2 = 900$$

Taking square root

$$\sqrt{m^2} = \pm\sqrt{900}$$

$$\boxed{m = \pm 30}$$

(ii) $20x^3y^5$, $5x^7y$

Solution:

Let mean proportional is m

then $20x^3y^5$, m , $5x^7y$

By proportion,

$20x^3y^5 : m : : m : 5x^7y$

Product of Means = Product of Extremes

$$m.m = (20x^3y^5)(5x^7y)$$

$$m^2 = 100x^{10}y^6$$

Taking square root of both sides

$$\sqrt{\cancel{m^2}} = \pm\sqrt{100x^{10}y^6}$$

$$m = \pm\sqrt{100}.\sqrt{x^{10}}.\sqrt{y^6}$$

$$m = \pm\sqrt{100}.\sqrt{x^{10}}.\sqrt{y^6}$$

$$m = \pm 10x^{10\times\frac{1}{2}}.y^{6\times\frac{1}{2}}$$

$$\boxed{m = \pm 10x^5y^3}$$

(iii) $15p^4qr^3, 135q^5r^7$

Solution:

Let mean proportional is m

then $15p^4qr^3$, m , $135q^5r^7$

By proportion

$15p^4qr^3 : m : : m : 135q^5r^7$

Product of Means = Product of Extremes

$$m.m = (15p^4qr^3)(135q^5r^7)$$

$$m^2 = 2025p^4q^6r^{10}$$

Taking square root

$$\sqrt{m\cancel{^2}} = \pm\sqrt{2025p^4q^6r^{10}}$$

$$m = \pm\sqrt{2025}\sqrt{p^4}.\sqrt{q^6}.\sqrt{r^{10}}$$

$$m = \pm 45p^{4\times\frac{1}{2}}.q^{6\times\frac{1}{2}}.r^{10\times\frac{1}{2}}$$

$$\boxed{m = \pm 45p^2.q^3.r^5}$$

(iv) $x^2 - y^2, \frac{x-y}{x+y}$

Solution:

Let mean proportional is m.

then $x^2 - y^2, m, \frac{x-y}{x+y}$

By proportion

$$x^2 - y^2 : m :: m : \frac{x-y}{x+y}$$

Product of Means = Product of Extremes

$$m.m = (x^2 - y^2)\frac{(x-y)}{x+y}$$

$$m^2 = \frac{\cancel{(x+y)}(x-y)(x-y)}{\cancel{(x+y)}}$$

$$m^2 = (x-y)^2$$

Taking square root

$$\sqrt{m^{\cancel{2}}} = \pm\sqrt{(x-y)^{\cancel{2}}}$$

$$\boxed{m = \pm(x-y)}$$

Q.4 Find the values of the letter involved in the following continued proportions

(i) 5, p, 45

Solution:

By continued proportion

$$5 : p :: p : 45$$

Product of Means = Product of Extremes

$$p.p = 5 \times 45$$

$$p^2 = 225$$

Taking square root of both sides

$$\sqrt{p^2} = \pm\sqrt{225}$$

$$\boxed{p = \pm 15}$$

(ii) 8, x, 18

Solution:

By continued proportion

$$8 : x :: x : 18$$

Product of Means = Product of Extremes

$$x.x = 8 \times 18$$

$$x^2 = 144$$

Taking square root

$$\sqrt{x^2} = \pm\sqrt{144}$$

$$\boxed{x = \pm 12}$$

(iii) 12, 3p – 6, 27

Solution:

By continued proportion

$$12 : 3p - 6 :: 3p - 6 : 27$$

Product of Means = Product of Extremes.

$$(3p-6)(3p-6) = 12 \times 27$$

$$(3p-6)^2 = 324$$

Taking square root of both sides

$$\sqrt{(3p-6)^2} = \pm\sqrt{324}$$

$$3p - 6 = \pm 18$$

$3p - 6 = 18$ or $3p - 6 = -18$

$3p = 18 + 6$ or $3p = -18 + 6$

$3p = 24$ or $3p = -12$

$p = \frac{24}{3}$ or $p = \frac{-12}{3}$

$p = 8$ or $p = -4$

(iv) $7, m-3, 28$

Solution:

By continued proportion

$$7 : m - 3 :: m - 3 : 28$$

Product of Means = Product of Extremes

$$(m-3)(m-3) = 7 \times 28$$

$$(m-3)^2 = 196$$

Taking square root of both sides

$$\sqrt{(m-3)^2} = \pm\sqrt{196}$$

$$m - 3 = \pm 14$$

$m - 3 = 14$ or $m - 3 = -14$

$m = 14 + 3$ or $m = -14 + 3$

$m = 17$ or $m = -11$