EXERCISE 3.4

Q.1 Prove that a : b = c : d , if

(i) $\dfrac{4a+5b}{4a-5b}=\dfrac{4c+5d}{4c-5d}$

(ii) $\dfrac{2a+9b}{2a-9b}=\dfrac{2c+9d}{2c-9d}$

(iii) $\dfrac{ac^2+bd^2}{ac^2-bd^2}=\dfrac{c^3+d^3}{c^3-d^3}$

(iv) $\dfrac{a^2c+b^2d}{a^2c-b^2d}=\dfrac{ac^2+bd^2}{ac^2-bd^2}$

(v) $pa+qb : pa-qb = pc+qd : pc-qd$

(vi) $\dfrac{a+b+c+d}{a+b-c-d}=\dfrac{a-b+c-d}{a-b-c+d}$

(vii) $\dfrac{2a+3b+2c+3d}{2a+3b-2c-3d}=\dfrac{2a-3b+2c-3d}{2a-3b-2c+3d}$

(viii) $\dfrac{a^2+b^2}{a^2-b^2}=\dfrac{ac+bd}{ac-bd}$

Solutions:

(i) $\dfrac{4a+5b}{4a-5b}=\dfrac{4c+5d}{4c-5d}$

Solution:

$$\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$$

By componendo – dividendo theorem

$$\frac{(4a+5b)+(4a-5b)}{(4a+5b)-(4a-5b)}=\frac{(4c+5d)+(4c-5d)}{(4c+5d)-(4c-5d)}$$

$$\frac{4a+\cancel{5b}+4a-\cancel{5b}}{\cancel{4a}+5b-\cancel{4a}+5b}=\frac{4c\cancel{+5d}+4c-\cancel{5d}}{\cancel{4c}+5d-\cancel{4c}+5d}$$

$$\frac{8a}{10b}=\frac{8c}{10d}$$

By multiplying both sides by $\dfrac{10}{8}$

$$\frac{\cancel{10}}{\cancel{8}}\frac{\cancel{8}}{\cancel{10}}\frac{a}{b}=\frac{\cancel{10}}{\cancel{8}}\frac{\cancel{8}c}{\cancel{10}d}$$

$\dfrac{a}{b}=\dfrac{c}{d} \Rightarrow a:b=c:d$ Hence proved

(ii) $\quad \mathbf{\dfrac{2a+9b}{2a-9b}=\dfrac{2c+9d}{2c-9d}}$

Solution:

$$\frac{2a+9b}{2a-9b}=\frac{2c+9d}{2c-9d}$$

By componendo – dividendo theorem

$$\frac{(2a+9b)+(2a-9b)}{(2a+9b)-(2a-9b)}=\frac{(2c+9d)+(2c-9d)}{(2c+9d)-(2c-9d)}$$

$$\frac{2a+\cancel{9b}+2a-\cancel{9b}}{\cancel{2a}+9b-\cancel{2a}+9b}=\frac{2c+\cancel{9d}+2c-\cancel{9d}}{\cancel{2c}+9d-\cancel{2c}+9d}$$

$$\frac{4a}{18b}=\frac{4c}{18d}$$

Multiplying both sides by $\dfrac{18}{4}$

$$\frac{\cancel{18}}{\cancel{4}}\times\frac{\cancel{4}\,a}{\cancel{18}\,b}=\frac{\cancel{4}\,c}{\cancel{18}\,d}.\frac{\cancel{18}}{\cancel{4}}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow \quad a:b=c:d$

Hence proved

(iii) $\quad \mathbf{\dfrac{ac^2+bd^2}{ac^2-bd^2}=\dfrac{c^3+d^3}{c^3-d^3}}$

Solution:

$$\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$$

By componendo – dividendo theorem

$$\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}=\frac{(c^3+d^3)+(c^3-d^3)}{(c^3+d^3)-(c^3-d^3)}$$

$$\frac{ac^2+\cancel{bd^2}+ac^2-\cancel{bd^2}}{\cancel{ac^2}+bd^2-\cancel{ac^2}+bd^2}=\frac{c^3+\cancel{d^3}+c^3-\cancel{d^3}}{\cancel{c^3}+d^3-\cancel{c^3}+d^3}$$

$$\frac{\cancel{2}ac^2}{\cancel{2}bd^2}=\frac{\cancel{2}c^3}{\cancel{2}d^3}$$

$$\frac{ac^2}{bd^2}=\frac{c.c^2}{d.d^2}$$

Multiplying both sides by $\dfrac{d^2}{c^2}$

$$\frac{a\,\cancel{c^2}}{b\,\cancel{d^2}}\times\frac{\cancel{d^2}}{\cancel{c^2}}=\frac{c\,\cancel{c^2}}{d\,\cancel{d^2}}\times\frac{\cancel{d^2}}{\cancel{c^2}}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow \quad a:b=c:d$ Hence proved

(iv) $\quad \mathbf{\dfrac{a^2c+b^2d}{a^2c-b^2d}=\dfrac{ac^2+bd^2}{ac^2-bd^2}}$

Solution:

$$\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$$

By componendo – dividendo theorem

$$\frac{(a^2c+b^2d)+(a^2c-b^2d)}{(a^2c+b^2d)-(a^2c-b^2d)}=\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}$$

$$\frac{a^2c+\cancel{b^2d}+a^2c-\cancel{b^2d}}{\cancel{a^2c}+b^2d-\cancel{a^2c}+b^2d}=\frac{ac^2+\cancel{bd^2}+ac^2-\cancel{bd^2}}{\cancel{ac^2}+bd^2-\cancel{ac^2}+bd^2}$$

$$\frac{\cancel{2}a^2c}{\cancel{2}b^2d}=\frac{\cancel{2}ac^2}{\cancel{2}bd^2}$$

$$\frac{a.ac}{b.bd}=\frac{ac.c}{bd.d}$$

Multiplying both sides by $\dfrac{bd}{ac}$

$$\frac{a.\cancel{ac}}{b.\cancel{bd}}\times\frac{\cancel{bd}}{\cancel{ac}}=\frac{\cancel{bd}}{\cancel{ac}}.\frac{\cancel{ac}.c}{\cancel{bd}.d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow \quad a:b=c:d$ Hence proved

(v) $\quad \mathbf{pa+qb:pa-qb=pc+qd:pc-qd}$

Solution:

$$pa+qb:pa-qb=pc+qd:pc-qd$$

OR

$$\frac{pa+qb}{pa-qb}=\frac{pc+qd}{pc-qd}$$

By componendo– dividendo theorem

$$\frac{(pa+qb)+(pa-qb)}{(pa+qb)-(pa-qb)}=\frac{(pc+qd)+(pc-qd)}{(pc+qd)-(pc-qd)}$$

$$\frac{pa+\cancel{qb}+pa-\cancel{qb}}{\cancel{pa}+qb-\cancel{pa}+qb}=\frac{pc+\cancel{qd}+pc-\cancel{qd}}{\cancel{pc}+qd-\cancel{pc}+qd}$$

$$\frac{\cancel{2}pa}{\cancel{2}qb}=\frac{\cancel{2}pc}{\cancel{2}qd}$$

$$\frac{pa}{qb}=\frac{pc}{qd}$$

Multiplying both sides by $\frac{q}{p}$

$$\frac{\cancel{q}}{\cancel{p}}\cdot\frac{\cancel{p}\,a}{\cancel{q}\,b}=\frac{\cancel{q}}{\cancel{p}}\cdot\frac{\cancel{p}\,c}{\cancel{q}\,d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$ Hence proved

(vi) $$\frac{\mathbf{a+b+c+d}}{\mathbf{a+b-c-d}}=\frac{\mathbf{a-b+c-d}}{\mathbf{a-b-c+d}}$$

Solution:

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

By componendo dividendo theorem

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{a+b+\cancel{c}+\cancel{d}+a+b-\cancel{c}-\cancel{d}}{\cancel{a}+\cancel{b}+c+d-\cancel{a}-\cancel{b}+c+d}=\frac{a-b+\cancel{c}-\cancel{d}+a-b-\cancel{c}+\cancel{d}}{\cancel{a}-\cancel{b}+c-d-\cancel{a}+\cancel{b}+c-d}$$

$$\frac{2a+2b}{2c+2d}=\frac{2a-2b}{2c-2d}$$

$$\frac{\cancel{2}(a+b)}{\cancel{2}(c+d)}=\frac{\cancel{2}(a-b)}{\cancel{2}(c-d)}$$

By alternendo theorem

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

Again by componendo– dividendo theorem

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)}=\frac{(c+d)+(c-d)}{(c-d)-(c-d)}$$

$$\frac{a+\cancel{b}+a-\cancel{b}}{\cancel{a}+b-\cancel{a}+b}=\frac{c+\cancel{d}+c-\cancel{d}}{\cancel{c}+d-\cancel{c}+d}$$

$$\frac{\cancel{2}a}{\cancel{2}b}=\frac{\cancel{2}c}{\cancel{2}d}\quad\Rightarrow\quad\frac{a}{b}=\frac{c}{d}\Rightarrow a:b=c:d$$

(vii) $$\frac{\mathbf{2a+3b+2c+3d}}{\mathbf{2a+3b-2c-3d}}=\frac{\mathbf{2a-3b+2c-3d}}{\mathbf{2a-3b-2c+3d}}$$

Solution:

$$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

By componendo–dividendo theorem

$$\frac{(2a+3b+2c+3d)+(2a+3b-2c-3d)}{(2a+3b+2c+3d)-(2a+3b-2c-3d)}$$

$$=\frac{(2a-3b+2c-3d)+(2a-3b-2c+3d)}{(2a-3b+2c-3d)-(2a-3b-2c+3d)}$$

$$\frac{2a+3b+\cancel{2c}+\cancel{3d}+2a+3b-\cancel{2c}-\cancel{3d}}{\cancel{2a}+\cancel{3b}+2c+3d-\cancel{2a}-\cancel{3b}+2c+3d}$$

$$=\frac{2a-3b+\cancel{2c}-\cancel{3d}+2a-3b-\cancel{2c}+\cancel{3d}}{\cancel{2a}-\cancel{3b}+2c-3d-\cancel{2a}+\cancel{3b}+2c-3d}$$

$$\frac{4a+6b}{4c+6d}=\frac{4a-6b}{4c-6d}$$

By alternando theorem

$$\frac{4a+6b}{4a-6b}=\frac{4c+6d}{4c-6d}$$

Again by componendo–dividendo theorem

$$\frac{(4a+6b)+(4a-6b)}{(4a+6b)-(4a-6b)}=\frac{(4c+6d)+(4c-6d)}{(4c+6d)-(4c-6d)}$$

$$\frac{4a+\cancel{6b}+4a-\cancel{6b}}{\cancel{4a}+6b-\cancel{4a}+6b}=\frac{4c+\cancel{6d}+4c-\cancel{6d}}{\cancel{4c}+6d-\cancel{4c}+6d}$$

$$\frac{8a}{12b}=\frac{8c}{12d}$$

Multiplying both sides by $\frac{12}{8}$

$$\frac{\cancel{8}a}{\cancel{12}b}\cdot\frac{\cancel{12}}{\cancel{8}}=\frac{\cancel{12}}{\cancel{8}}\cdot\frac{\cancel{8}c}{\cancel{12}d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:d=c:d$ Hence proved

(viii) $$\frac{\mathbf{a^2+b^2}}{\mathbf{a^2-b^2}}=\frac{\mathbf{ac+bd}}{\mathbf{ac-bd}}$$

Solution:

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

By componendo – dividendo theorem

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)} = \frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+\cancel{b^2}+a^2-\cancel{b^2}}{\cancel{a^2}+b^2-\cancel{a^2}+b^2} = \frac{ac+\cancel{bd}+ac-\cancel{bd}}{\cancel{ac}+bd-\cancel{ac}+bd}$$

$$\frac{\cancel{2}a^2}{\cancel{2}b^2} = \frac{\cancel{2}ac}{\cancel{2}bd}$$

$$\frac{a.a}{b.a} = \frac{ac}{bd}$$

Multiplying both sides by "$\frac{b}{a}$"

$$\frac{\cancel{b}}{\cancel{a}}.\frac{\cancel{a}.a}{\cancel{b}.a} = \frac{\cancel{b}}{\cancel{a}}\frac{\cancel{a}c}{\cancel{b}d}$$

$$\frac{a}{b} = \frac{c}{d}$$

$\Rightarrow$ $a : b = c : d$ Hence proved

Q.2 Use componendo–dividendo theorem to find the values of the following.

(i) Find the value of

$$\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z}, \text{ if } x = \frac{4yz}{y+z}$$

Solution: $x = \frac{4yz}{y+z}$ .... (i)

Dividing the equation (i) by "2y"

$$\frac{x}{2y} = \frac{4yz}{(y+z).2y}$$

$$\frac{x}{2y} = \frac{2z}{y+z}$$

By componendo – dividendo theorem

$$\frac{x+2y}{x-2y} = \frac{2z+(y+z)}{2z-(y+z)}$$

$$\frac{x+2y}{x-2y} = \frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y} = \frac{3z+y}{z-y} \quad \text{................ (ii)}$$

Now dividing the eq (i) by "2z"

$$\frac{x}{2z} = \frac{4yz}{(y+z).2z}$$

$$\frac{x}{2z} = \frac{2y}{y+z}$$

By componendo – dividendo theorem

$$\frac{x+2z}{x-2z} = \frac{2y+(y+z)}{2y-(y+z)}$$

$$\frac{x+2z}{x-2z} = \frac{2y+y+z}{2y-y-z}$$

$$\frac{x+2z}{x-2z} = \frac{3y+z}{y-z} \quad \text{................ (iii)}$$

Adding eq. (ii) and (iii)

$$\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z} = \frac{3z+y}{z-y} + \frac{3y+z}{y-z}$$

$$= \frac{3z+y}{-1(y-z)} + \frac{3y+z}{y-z}$$

$$= \frac{-1(3z+y)}{y-z} + \frac{3y+z}{y-z}$$

$$= \frac{-3z-y+3y+z}{y-z}$$

$$= \frac{2y-2z}{y-z}$$

$$= \frac{2\cancel{(y-z)}}{\cancel{(y-z)}}$$

$$\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z} = 2$$

(ii) Find the value of

$$\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p}, \text{ if } m = \frac{10np}{n+p}$$

Solution: $m = \frac{10np}{n+p}$ .... (i)

Dividing equation (i) by "5n"

$$\frac{m}{5n} = \frac{10np}{(n+p)5n}$$

$$\frac{m}{5n} = \frac{2p}{n+p}$$

By componendo – dividendo theorem

$$\frac{m+5n}{m-5n}=\frac{2p+(n+p)}{2p-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2p+n+p}{2p-n-p}$$

$$\frac{m+5n}{m-5n}=\frac{3p+n}{p-n} \quad \text{................ (ii)}$$

Now, dividing equation (i) by "5p"

$$\frac{m}{5p}=\frac{10np}{(n+p)5p}$$

$$\frac{m}{5p}=\frac{2n}{n+p}$$

By componendo– dividendo theorem

$$\frac{m+5p}{m-5p}=\frac{2n+(n+p)}{2n-(n+p)}$$

$$\frac{m+5p}{m-5p}=\frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5p}{m-5p}=\frac{3n+p}{n-p} \quad \text{.............. (iii)}$$

Adding equation (ii) and (iii)

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{3n+p}{n-p}$$

$$=\frac{3p+n}{-1(n-p)}+\frac{3n+p}{n-p}$$

$$=\frac{-(3p+n)}{n-p}+\frac{3n+p}{n-p}$$

$$=\frac{-3p-n+3n+p}{n-p}$$

$$=\frac{2n-2p}{n-p}$$

$$=\frac{2(n-p)}{(n-p)}$$

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=2$$

(iii) Find the value of $\mathbf{\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}}$ **if** $\mathbf{x=\frac{12ab}{a-b}}$

Solution: $x=\frac{12ab}{a-b}$ .... (i)

Dividing equation (i) by 6a

$$\frac{x}{6a}=\frac{12ab}{(a-b).6a}$$

$$\frac{x}{6a}=\frac{2b}{a-b}$$

By componendo– dividendo theorem

$$\frac{x+6a}{x-6a}=\frac{2b+(a-b)}{2b-(a-b)}$$

$$\frac{x+6a}{x-6a}=\frac{2b+a-b}{2b-a+b}$$

$$\frac{x+6a}{x-6a}=\frac{a+b}{3b-a}$$

By invertendo theorem

$$\frac{x-6a}{x+6a}=\frac{3b-a}{a+b} \quad \text{................ (i)}$$

Now, dividing the equation (i) by 6b.

$$\frac{x}{6b}=\frac{12ab}{(a-b).6b}$$

$$\frac{x}{6b}=\frac{2a}{a-b}$$

By componendo– dividendo theorem

$$\frac{x+6b}{x-6b}=\frac{2a+(a-b)}{2a-(a-b)}$$

$$\frac{x+6b}{x-6b}=\frac{2a+a-b}{2a-a+b}$$

$$\frac{x+6b}{x-6b}=\frac{3a-b}{a+b} \quad \text{................ (ii)}$$

Subtracting equation (iii) from (ii)

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{3b-a}{a+b}-\frac{3a-b}{a+b}$$

$$=\frac{(3b-a)-(3a-b)}{a+b}$$

$$=\frac{3b-a-3a+b}{a+b}$$

$$= \frac{4b - 4a}{a + b}$$

$$= \frac{4(b - a)}{(a + b)}$$

(iv) **Find the value of**

$$\frac{x - 3y}{x + 3y} - \frac{x + 3z}{x - 3z}, \text{ if } x = \frac{3yz}{y - z}$$

Solution: $x = \frac{3yz}{y - z}$ .... (i)

Dividing equation (i) by "3y"

$$\frac{x}{3y} = \frac{3yz}{(y - z).3y}$$

$$\frac{x}{3y} = \frac{z}{y - z}$$

By componendo – dividendo theorem

$$\frac{x + 3y}{x - 3y} = \frac{z + (y - z)}{z - (y - z)}$$

$$\frac{x + 3y}{x - 3y} = \frac{\cancel{z} + y - \cancel{z}}{z - y + z}$$

$$\frac{x + 3y}{x - 3y} = \frac{y}{2z - y}$$

By invertendo theorem

$$\frac{x - 3y}{x + 3y} = \frac{2z - y}{y} \text{ .... (ii)}$$

Now dividing equation (i) by "3z"

$$\frac{x}{3z} = \frac{3yz}{(y - z).3z}$$

$$\frac{x}{3z} = \frac{y}{y - z}$$

By componendo–dividendo theorem

$$\frac{x + 3z}{x - 3z} = \frac{y + (y - z)}{y - (y - z)}$$

$$\frac{x + 3z}{x - 3z} = \frac{y + y - z}{\cancel{y} - \cancel{y} + z}$$

$$\frac{x + 3z}{x - 3z} = \frac{2y - z}{z} \text{ .... (iii)}$$

Subtracting equation (iii) from (ii)

$$\frac{x - 3y}{x + 3y} - \frac{x + 3z}{x - 3z} = \frac{2z - y}{y} - \frac{2y - z}{z}$$

$$= \frac{z(2z - y) - y(2y - z)}{yz}$$

$$= \frac{2z^2 - \cancel{yz} - 2y^2 + \cancel{yz}}{yz}$$

$$= \frac{2z^2 - 2y^2}{yz} = \frac{2(z^2 - y^2)}{yz}$$

(v) **Find the value of**

$$\frac{s - 3p}{s + 3p} + \frac{s + 3q}{s - 3q}, \text{ if } s = \frac{6pq}{p - q}$$

Solution:

$s = \frac{6pq}{p - q}$ ....(i)

Dividing equation (i) by "3P"

$$\frac{s}{3p} = \frac{6pq}{(p - q).3p}$$

$$\frac{s}{3p} = \frac{2q}{p - q}$$

By componendo–dividendo theorem

$$\frac{s + 3p}{s - 3p} = \frac{2q + (p - q)}{2q - (p - q)}$$

$$\frac{s + 3p}{s - 3p} = \frac{2q + p - q}{2q - p + q}$$

$$\frac{s + 3p}{s - 3p} = \frac{p + q}{3q - p}$$

By invertendo theorem

$$\frac{s - 3p}{s + 3p} = \frac{3q - p}{p + q} \text{ .... (ii)}$$

Now dividing equation (i) by "3q"

$$\frac{s}{3q} = \frac{2\cancel{6}p\cancel{q}}{(p - q).\cancel{3}\cancel{q}}$$

$$\frac{s}{3q} = \frac{2p}{p - q}$$

By componendo – dividendo theorem

$$\frac{s + 3q}{s - 3q} = \frac{2p + (p - q)}{2p - (p - q)}$$

$$\frac{s + 3q}{s - 3q} = \frac{2p + p - q}{2p - p + q}$$

$$\frac{s+3q}{s-3q}=\frac{3p-q}{p+q}\text{.......................}(iii)$$

Adding equation (ii) and (iii)

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p}{p+q}+\frac{3p-q}{p+q}$$

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p+3p-q}{(p+q)}$$

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{2p+2q}{(p+q)}$$

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{2\cancel{(p+q)}}{\cancel{(p+q)}}$$

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=2$$

(vi) Solve $\dfrac{(\mathbf{x-2})^2-(\mathbf{x-4})^2}{(\mathbf{x-2})^2+(\mathbf{x-4})^2}=\dfrac{\mathbf{12}}{\mathbf{13}}$

Solution:

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$$

By componendo–dividendo theorem

$$\frac{[(x-2)^2-(x-4)^2]+[(x-2)^2+(x-4)^2]}{[(x-2)^2-(x-4)^2]-[(x-2)^2+(x-4)^2]}=\frac{12+13}{12-13}$$

$$\frac{(x-2)^2-\cancel{(x-4)^2}+(x-2)^2+\cancel{(x-4)^2}}{\cancel{(x-2)^2}-(x-4)^2-\cancel{(x-2)^2}-(x-4)^2}=\frac{25}{-1}$$

$$\frac{\cancel{2}(x-2)^2}{\cancel{-}\cancel{2}(x-4)^2}=\cancel{-}25$$

$$\left(\frac{x-2}{x-4}\right)^2=25$$

Taking square root of both side

$$\sqrt{\left(\frac{x-2}{x-4}\right)^2}=\pm\sqrt{25}$$

$$\frac{x-2}{x-4}=\pm5$$

$$\frac{x-2}{x-4}=5 \quad \text{or} \quad \frac{x-2}{x-4}=-5$$

$x-2=5(x-4)$ or $x-2=-5(x-4)$

$x-2=5x-20$ or $x-2=-5x+20$

$20-2=5x-x$ or $x+5x=20+2$

$18=4x$ or $6x=22$

$$\frac{\cancel{18}^{\,9}}{\cancel{4}_{\,2}}=x \quad \text{or} \quad x=\frac{22}{6}$$

$$x=\frac{9}{2} \quad \text{or} \quad x=\frac{11}{3}$$

$$S.S=\left\{\frac{9}{2},\frac{11}{3}\right\}$$

(vii) Solve $\dfrac{\sqrt{\mathbf{x^2+2}}+\sqrt{\mathbf{x^2-2}}}{\sqrt{\mathbf{x^2+2}}-\sqrt{\mathbf{x^2-2}}}=\dfrac{\mathbf{2}}{\mathbf{1}}$

Solution:

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=\frac{2}{1}$$

By componendo– dividendo theorem

$$\frac{\left(\sqrt{x^2+2}+\sqrt{x^2-2}\right)+\left(\sqrt{x^2+2}-\sqrt{x^2-2}\right)}{\left(\sqrt{x^2+2}+\sqrt{x^2-2}\right)-\left(\sqrt{x^2+2}-\sqrt{x^2-2}\right)}=\frac{2+1}{2-1}$$

$$\frac{\sqrt{x^2+2}+\cancel{\sqrt{x^2-2}}+\sqrt{x^2+2}-\cancel{\sqrt{x^2-2}}}{\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}-\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}}=\frac{3}{1}$$

$$\frac{\cancel{2}\sqrt{x^2+2}}{\cancel{2}\sqrt{x^2-2}}=3$$

$$\sqrt{\frac{x^2+2}{x^2-2}}=3$$

Taking square of both sides

$$\left(\sqrt{\frac{x^2+2}{x^2-2}}\right)^{\cancel{2}}=(3)^2$$

$$\frac{x^2+2}{x^2-2}=9$$

$x^2+2=9(x^2-2)$

$x^2+2=9x^2-18$

$2+18=9x^2-x^2$

$20=8x^2$

$$x^2=\frac{\cancel{20}^{\,5}}{\cancel{8}_{\,2}} \quad \Rightarrow \quad x^2=\frac{5}{2}$$

Taking square root

$$\sqrt{x^2}=\pm\sqrt{\frac{5}{2}} \quad \Rightarrow \quad x=\pm\sqrt{\frac{5}{2}}$$

$$S.S=\left\{\pm\sqrt{\frac{5}{2}}\right\}$$

(viii) **Solve** $\dfrac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}=\dfrac{1}{3}$

Solution:

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}=\frac{1}{3}$$

By componendo–dividendo theorem

$$\frac{\left(\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}\right)+\left(\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}\right)}{\left(\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}\right)-\left(\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}\right)}=\frac{1+3}{1-3}$$

$$\frac{\sqrt{x^2+8p^2}-\cancel{\sqrt{x^2-p^2}}+\sqrt{x^2+8p^2}+\cancel{\sqrt{x^2-p^2}}}{\cancel{\sqrt{x^2+8p^2}}-\sqrt{x^2-p^2}-\cancel{\sqrt{x^2+8p^2}}-\sqrt{x^2-p^2}}=\frac{4}{-2}$$

$$\frac{\cancel{2}\sqrt{x^2+8p^2}}{\cancel{-}\cancel{2}\sqrt{x^2-p^2}}=\cancel{-}2$$

$$\sqrt{\frac{x^2+8p^2}{x^2-p^2}}=2$$

Taking square of both sides

$$\left(\sqrt{\frac{x^2+8p^2}{x^2-p^2}}\right)^2=(2)^2$$

$$\frac{x^2+8p^2}{x^2-p^2}=4$$

$x^2 + 8p^2 = 4(x^2 - p^2)$

$x^2 + 8p^2 = 4x^2 - 4p^2$

$8p^2 + 4p^2 = 4x^2 - x^2$

$12p^2 = 3x^2$

$\Rightarrow$ $3x^2 = 12p^2$

$$x^2=\frac{12p^2}{3}$$

$x^2 = 4p^2$

Taking square root

$$\sqrt{x^2}=\pm\sqrt{4p^2}$$

$x = \pm 2p$

S.S = $\{\pm 2p\}$

(ix) **Solve** $\dfrac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\dfrac{13}{14}$

Solution:

$$\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\frac{13}{14}$$

By componendo–dividendo theorem

$$\frac{\left[(x+5)^3-(x-3)^3\right]+\left[(x+5)^3+(x-3)^3\right]}{\left[(x+5)^3-(x-3)^3\right]-\left[(x+5)^3+(x-3)^3\right]}=\frac{13+14}{13-14}$$

$$\frac{(x+5)^3-\cancel{(x-3)^3}+(x+5)^3+\cancel{(x-3)^3}}{\cancel{(x+5)^3}-(x-3)^3-\cancel{(x+5)^3}-(x-3)^3}=\frac{27}{-1}$$

$$\frac{\cancel{2}(x+5)^3}{\cancel{-}\cancel{2}(x-3)^3}=\cancel{-}27$$

$$\left(\frac{x+5}{x-3}\right)^3=27$$

Taking cube root

$$\sqrt[3]{\left(\frac{x+5}{x-3}\right)^3}=\sqrt[3]{27}$$

$$\left(\frac{x+5}{x-3}\right)^{\cancel{3}\times\frac{1}{\cancel{3}}}=(3)^{\cancel{3}\times\frac{1}{\cancel{3}}}$$

$$\frac{x+5}{x-3}=3$$

$(x+5) = 3(x - 3)$

$x + 5 = 3x - 9$

$5 + 9 = 3x - x$

$14 = 2x$

$$\frac{14}{2}=x$$

$7 = x$

$\Rightarrow$ $x = 7$

S.S = $\{7\}$