EXERCISE 3.5

Q.1 If s varies directly as u^2 and inversely as v and s = 7, when u = 3, v = 2. Find the value of s when u = 6 and v = 10

Solution: $\boxed{s \propto \frac{u^2}{v}}$ $\boxed{s = 7, u = 3, v = 2}$

$\boxed{s = ?, u = 6, v = 10}$

$$s \propto \frac{u^2}{v}$$

$$s = \frac{ku^2}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put s = 7 = , u = 3 , v = 2 in equation (i)

$$7 = \frac{k(3)^2}{2}$$

$$7 \times 2 = k(9)$$

$$\frac{14}{9} = k \qquad \Rightarrow \qquad k = \frac{14}{9}$$

Put $k = \frac{14}{9}$ in equation (i)

$$s = \frac{14u^2}{9v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

To find s,

Now put, u=6 and v=10 in equation (i)

$$s = \frac{14u^2}{9v}$$

$$s = \frac{14(6)^2}{9(10)}$$

$$s = \frac{14 \times \cancel{36}^{2}}{\cancel{90}_{5}} \qquad \text{(dividing by 18)}$$

$$s = \frac{14 \times 2}{5}$$

$$\boxed{s = \frac{28}{5}}$$

Q.2 If w varies jointly as x, y^2 and z and w = 5 when x = 2, y = 3, z = 10. Find w when x = 4, y = 7 and z = 3.

Solution:

$\boxed{w \propto xy^2z}$ $\boxed{w = 5, \ x = 2 \\ y = 3, \ z = 10}$ $\boxed{w = ?, x = 4 \\ y = 7, z = 3}$

$$w \propto xy^2z$$

$$w = kxy^2z \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put w =5, x = 2, y = 3 and z =10 in equation (i)

$$5 = k(2)(3)^2(10)$$

$$5 = k(20)(9)$$

$$5 = k(180)$$

$$\frac{5}{180} = k$$

$$\Rightarrow \qquad k = \frac{\cancel{5}^{1}}{\cancel{180}_{36}}$$

$$k = \frac{1}{36}$$

Put $k = \frac{1}{36}$ in equation (i)

$$w = \frac{1}{36}xy^2z \ldots\ldots\ldots\ldots (ii)$$

To find w,

Now put x = 4, y = 7, z = 3 in equation (ii)

$$w = \frac{1}{36}xy^2z$$

$$w = \frac{1}{36}(4)(7)^2(3)$$

$$w = \frac{1}{36}(4)(49)(3)$$

$$w = \frac{1}{\cancel{36}_{3}}(49) \times \cancel{12}^{1}$$

$$w = \frac{1}{3}(49)$$

$$\boxed{w = \frac{49}{3}}$$

Q.3 If y varies directly as x^3 and inversely as z^2 and t, and y = 16 when x =4, z = 2, t = 3. Find the value of y when x = 2, z = 3 and t = 4.

Solution:

$$\boxed{y \propto \frac{x^3}{z^2t}} \quad \boxed{y = 16, x = 4 \\ z = 2, t = 3} \quad \boxed{y = ?, x = 2 \\ z = 3, t = 4}$$

$$y \propto \frac{x^3}{z^2t}$$

$$y = \frac{kx^3}{z^2t} \qquad \text{.....(i)}$$

To find k,

Put y =16,x = 4,z = 2 and t = 3 in equation (i)

$$16 = \frac{k(4)^3}{(2)^2(3)}$$

$$16 = \frac{k(64)}{4\times3}$$

$$\frac{16\times4\times3}{64} = k$$

$$\frac{\cancel{64}}{\cancel{64}}\times3 = k \quad \Rightarrow \quad k = 3$$

Put k = 3, in equation (i)

$$y = \frac{3x^3}{z^2t} \text{.....(ii)}$$

To find y,

Now, put x = 2, z = 3 and t = 4 in equation (ii)

$$y = \frac{3\times(2)^3}{(3)^2(4)}$$

$$y = \frac{3\times8}{9\times4}$$

$$y = \frac{24}{36}$$

$$y = \frac{2}{3}\cancel{\frac{24}{36}} \quad (\textit{dividing by } 12)$$

$$\boxed{y = \frac{2}{3}}$$

Q.4 If u varies directly as x^2 and inversely as product yz^3, and u=2, when x = 8, y = 7, z = 2. Find the value of u when x =6,y=3,z = 2.

Solution:

$$\boxed{u \propto \frac{x^2}{yz^3}} \quad \boxed{u = 2, x = 8 \\ y = 7, z = 2} \quad \boxed{u = ?, x = 6 \\ y = 3, z = 2}$$

$$u \propto \frac{x^2}{yz^3}$$

$$u = \frac{kx^2}{yz^3} \qquad \text{.....(i)}$$

To find k,

Put u =2, x = 8, y = 7 and z = 2 in equation(i)

$$2 = \frac{k(8)^2}{7(2)^3}$$

$$2 = \frac{k64}{7(8)}$$

$$\frac{2(8)(7)}{64} = k$$

$$\frac{\cancel{16}^{1}(7)}{\cancel{64}\,4} = k$$

$$\frac{7}{4} = k \quad \Rightarrow \quad k = \frac{7}{4}$$

Put $k = \frac{7}{4}$ in equation (i)

$$u = \frac{7x^2}{4yz^3} \text{.....(ii)}$$

To find u,

Now put x = 6, y =3, z = 2 in equation (ii)

$$u = \frac{7x^2}{4yz^3} = \frac{7(6)^2}{4(3)(2)^3}$$

$$u = \frac{7(\cancel{36}^{3})}{\cancel{12}\times8}$$

$$u = \frac{7(3)}{8}$$

$$\boxed{u = \frac{21}{8}}$$

Q.5 If v varies directly as the product xy^3 and inversely as z^2 and v = 27 when x =7, y = 6, z = 7. Find the value of V when x =6; y =2, z =3.

Solution: $\boxed{v \propto \frac{xy^3}{z^2}}$ $\boxed{\begin{matrix} v = 27, x = 7 \\ y = 6, z = 7 \end{matrix}}$ $\boxed{\begin{matrix} v = ?, x = 6 \\ y = 2, z = 3 \end{matrix}}$

$$v \propto \frac{xy^3}{z^2}$$

$$v = \frac{kxy^3}{z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put v = 27, x = 7, y =6, z = 7 in equation(i)

$$27 = \frac{k(7)(6)^3}{(7)^2}$$

$$27 = \frac{k(216)}{7}$$

$$\frac{1\cancel{27} \times 7}{8\cancel{216}} = k$$

$$\Rightarrow \quad k = \frac{7}{8}$$

Put $k = \frac{7}{8}$ in equation (i)

$$v = \frac{7xy^3}{8z^2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

To find v,

Now, put x =6, y =2, z = 3 in equation (ii).

$$v = \frac{7}{8}\frac{(6)(2)^3}{(3)^2}$$

$$v = \frac{7}{\cancel{8}}\frac{(6)(\cancel{8})}{9}$$

$$v = \frac{\cancel{42}^{14}}{\cancel{9}_3}$$

$$\Rightarrow \quad \boxed{v = \frac{14}{3}}$$

Q.6 If w varies inversely as the cube of U, and w = 5 when U =3. Find w, when U = 6.

Solution: $\boxed{w \propto \frac{1}{u^3}}$ $\boxed{\begin{matrix} w = 5 \\ u = 3 \end{matrix}}$ $\boxed{\begin{matrix} w = ? \\ u = 6 \end{matrix}}$

$$w \propto \frac{1}{u^3}$$

$$w = \frac{k}{u^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put w = 5 and u = 3 in equation (i)

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$\Rightarrow \quad k = 27 \times 5$$

$$k = 135$$

Put k = 135 in equation (i)

$$w = \frac{k}{u^3}$$

$$w = \frac{135}{u^3} \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

To find w,

Now , Put u = 6 in equation (ii)

$$w = \frac{135}{(6)^3}$$

$$w = \frac{135}{216}$$

$$w = \frac{5 \times \cancel{27}}{8 \times \cancel{27}}$$

$$\boxed{w = \frac{5}{8}}$$