EXERCISE 3.6

Q.1 If a:b=c:d, (a,b,c,d ≠0), then show that

(i) $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

(ii) $\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}$

(iii) $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

(iv) $a^6+c^6 : b^6+d^6 = a^3c^3 : b^3d^3$

(v) $p(a+b)+qb : p(c+d)+qd = a : c$

(vi) $a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$

(vii) $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Solution:

(i) $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

Let $a : b = c : d = k$

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k, \text{ and } \frac{c}{d}=k$$

$\boxed{a = bk}$ and $\boxed{c = dk}$

$$L.H.S=\frac{4a-9b}{4a+9b}$$

$$=\frac{4bk-9b}{4bk+9b}$$

$$=\frac{b(4k-9)}{b(4k+9)}$$

$$L.H.S=\frac{4k-9}{4k+9} \qquad \text{.............. (i)}$$

Now , taking

$$R.H.S=\frac{4c-9d}{4c+9d}$$

$$=\frac{4dk-9d}{4dk+9d}$$

$$=\frac{d(4k-9)}{d(4k+d)}$$

$$R.H.S=\frac{4k-9}{4k+9} \qquad \text{.............. (ii)}$$

From equation (i) and (ii)

L.H.S = R.H.S

So $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

Hence proved

(ii) $\mathbf{\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}}$

Let a : b = c : d = k

$\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \frac{a}{b}=k \quad : \quad \frac{c}{d}=k$

$\boxed{a=bk}; \quad \boxed{c=dk}$

Let $L.H.S=\frac{6a-5b}{6a+5b}$

$=\frac{6bk-5b}{6bk+5b}$

$=\frac{b(6k-5)}{b(6k+5)}$

$L.H.S=\frac{6k-5}{6k+5}$ .... (i)

Now $R.H.S=\frac{6c-5d}{6c+5d}$

$=\frac{6dk-5d}{6dk+5d}$

$=\frac{d(6k-5)}{d(6k+5)}$

$R.H.S=\frac{6k-5}{6k+5}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So, $\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}$

Hence proved

(iii) $\mathbf{\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}}$

Let a : b = c : d = k

$\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k \quad : \quad \frac{c}{d}=k$

$\boxed{a=bk}; \quad \boxed{c=dk}$

Let $L.H.S=\frac{a}{b} = \frac{\cancel{b}k}{\cancel{b}}$

L.H.S = k ———————— (i)

Now $R.H.S=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

$=\sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$

$=\sqrt{\frac{k^2\cancel{(b^2+d^2)}}{\cancel{(b^2+d^2)}}}$

$=\sqrt{k^2}$

$R.H.S=k$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

Hence proved

So $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

(iv) $\mathbf{a^6+c^6:b^6+d^6 = a^3c^3 : b^3d^3}$

Let a : b = c : d = k

$\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k; \frac{c}{d}=k$

Let L.H.S = $a^6+c^6 : b^6+d^6$

$=\frac{a^6+c^6}{b^6+d^6} = \frac{(bk)^6+(dk)^6}{b^6+d^6}$

$=\frac{b^6k^6+d^6k^6}{b^6+d^6} = \frac{k^6\cancel{(b^6+d^6)}}{\cancel{(b^6+d^6)}}$

L.H.S = k^6 .... (i)

Now, R.H.S = $a^3c^3 : b^3d^3$

$=\frac{a^3c^3}{b^3d^3} = \frac{(bk)^3.(dk)^3}{b^3d^3}$

$=\frac{b^3k^3.d^3k^3}{b^3d^3} = \frac{\cancel{b^3d^3}.k^{3+3}}{\cancel{b^3d^3}}$

R.H.S = k^6 .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So $a^6+c^6 : b^6+d^6 = a^3c^3 : b^3d^3$

Hence proved

(v) $\mathbf{p\,(a+b)\ +qb : p\,(c+d)\ +qd = a : c}$

Let $a : b = c : d = k$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k \ ; \ \frac{c}{d} = k$$

$$\boxed{a = bk} \ ; \ \boxed{c = dk}$$

Let $L.H.S = p\left(a+b\right)+qb : p(c+d)+qd$

$$= \frac{p\left(a+b\right)+qb}{p\left(c+d\right)+qd}$$

$$= \frac{p\left(bk+b\right)+qb}{p\left(dk+d\right)+qd}$$

$$= \frac{pb\left(k+1\right)+qb}{pd\left(k+1\right)+qd}$$

$$= \frac{b\left[p\left(k+1\right)+q\right]}{d\left[p\left(k+1\right)+q\right]}$$

$$\text{L.H.S} = \frac{b}{d} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\text{R.H.S} = a : c = \frac{a}{c}$$

$$R.H.S = \frac{bk}{dk}$$

$$R.H.S = \frac{b}{d} \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

$$p\left(a+b\right)+qb : p\left(c+d\right)+qd = a : b$$

Hence proved

(vi) $\mathbf{a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}}$

Let $a : b = c : d = k$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k \ ; \ \frac{c}{d} = k$$

$$\boxed{a = bk} \ ; \ \boxed{c = dk}$$

Let $L.H.S = a^2+b^2 : \frac{a^3}{a+b}$

$$= \left[(bk)^2+b^2\right] \div \frac{(bk)^3}{bk+b}$$

$$= \left(b^2k^2+b^2\right) \times \frac{\left(bk+b\right)}{b^3k^3}$$

$$= b^2\left(k^2+1\right) \times \frac{b\left(k+1\right)}{b^3k^3}$$

$$= \frac{b^3\left(k^2+1\right)\left(k+1\right)}{b^3k^3}$$

$$L.H.S = \frac{\left(k^2+1\right)\left(k+1\right)}{k^3} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now $R.H.S = c^2+d^2 : \frac{c^3}{c+d}$

$$= \left[(dk)^2+d^2\right] \div \frac{(dk)^3}{\left(dk+d\right)}$$

$$= (d^2k^2+d^2) \times \frac{(dk+d)}{d^3k^3}$$

$$= d^2\left(k^2+1\right) \times \frac{d\left(k+1\right)}{d^3k^3}$$

$$= \frac{d^3\left(k^2+1\right)\left(k+1\right)}{d^3k^3}$$

$$R.H.S = \frac{\left(k^2+1\right)\left(k+1\right)}{k^3} \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

So $a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$

Hence proved

(vii) $\mathbf{\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}}$

Let $a : b = c : d = k$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k, \quad \frac{c}{d} = k$$

$$\boxed{a = bk}, \quad \boxed{c = dk}$$

Let $L.H.S = \frac{a}{a-b} : \frac{a+b}{b}$

$= \frac{bk}{bk-b} \div \frac{bk+b}{b}$

$= \frac{bk}{b(k-1)} \times \frac{b}{b(k+1)}$

$= \frac{k}{(k)^2-(1)^2}$

$L.H.S = \frac{k}{k^2-1}$ .... (i)

Now $R.H.S = \frac{c}{c-d} : \frac{c+d}{d}$

$= \frac{dk}{dk-d} \div \frac{dk+d}{d}$

$= \frac{dk}{d(k-1)} \times \frac{d}{d(k+1)}$

$= \frac{k}{(k)^2-(1)^2}$

$R.H.S = \frac{k}{k^2-1}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Hence proved

Q.2 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} (a, b, c, d, e, f \neq 0)$, then show that

(i) $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

(ii) $\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{2/3}$

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Solution:

(i) $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\frac{a}{b} = k$, $\frac{c}{d} = k$, $\frac{e}{f} = k$

$a = bk$, $c = dk$, $e = fk$

$L.H.S = \frac{a}{b} = \frac{bk}{b} = k$

L.H.S = k .... (i)

Now $R.H.S = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

$= \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$

$= \sqrt{\frac{k^2(b^2+d^2+f^2)}{(b^2+d^2+f^2)}}$

$= \sqrt{k^2}$

R.H.S = k .... (ii)

From (i) and (ii)

R.H.S = R.H.S

i.e $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

(ii) $\frac{ac+ce+ea}{bd+df+fb} = \left(\frac{ace}{bdf}\right)^{\frac{2}{3}}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\frac{a}{b} = k$, $\frac{c}{d} = k$, $\frac{e}{f} = k$

$a = bk;$ $c = dk,$ $e = fk$

Let $L.H.S = \frac{ac+ce+ea}{bd+df+fb}$

$= \frac{bk(dk)+dk(fk)+fk(bk)}{bd+df+fb}$

$= \frac{k^2bd+k^2df+k^2fb}{bd+df+fb}$

$= \frac{k^2(bd+df+fb)}{(bd+df+fb)}$

$L.H.S = k^2$ .... (i)

Now, $R.H.S = \left(\frac{ace}{bdf}\right)^{\frac{2}{3}}$

$$= \left(\frac{bk.dk.fk}{bdf}\right)^{\frac{2}{3}} = \left(\frac{k^3.bdf}{bdf}\right)^{\frac{2}{3}}$$

$$= \left(k^3\right)^{\frac{2}{3}} = k^{3\times\frac{2}{3}}$$

R.H.S = k^2 .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So, $\frac{ac+ce+ea}{bd+df+fb} = \left(\frac{ace}{bdf}\right)^{\frac{2}{3}}$

Hence proved.

(iii) $\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb} = \frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$$

$$\boxed{a = bk, \quad c = dk, \quad e = fk}$$

Let $L.H.S = \frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb}$

$$= \frac{bk.dk}{bd}+\frac{dk.fk}{df}+\frac{fk.bk}{fb}$$

$$= \frac{k^2\cancel{bd}}{\cancel{bd}}+\frac{k^2\cancel{df}}{\cancel{df}}+\frac{k^2\cancel{fb}}{\cancel{fb}}$$

$= k^2+k^2+k^2$

L.H.S = $3k^2$ .... (i) Now

$$R.H.S = \frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$$

$$= \frac{\cancel{b^2}k^2}{\cancel{b^2}}+\frac{\cancel{d^2}k^2}{\cancel{d^2}}+\frac{\cancel{f^2}k^2}{\cancel{f^2}}$$

$$= k^2+k^2+k^2$$

R.H.S $= 3k^2$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

$$\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{df} = \frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$$