

EXERCISE 3.7

Q. 1 The surface area A of a cube varies directly as the square of the length ℓ of an edge and $A = 27$ square units when $\ell = 3$ units. Find (i) A when $\ell = 4$ units (ii) ℓ when $A = 12$ sq. units.

Solution: $A \propto \ell^2$ $A = 27$ units, $\ell = 3$ units

$$(i) \quad A = ?, \ell = 4 \text{ units}$$

$$(ii) \quad \ell = ?, A = 12 \text{ units}$$

$$A \propto \ell^2$$

$$A = k\ell^2 \dots\dots\dots (i)$$

To find k,

Put $A = 27$ and $\ell = 3$ in equation (i)

$$27 = k(3)^2$$

$$27 = k(9)$$

$$\frac{27}{9} = k$$

$$3 = k$$

$$\Rightarrow \boxed{k = 3}$$

(i) To find A,

Put $k = 3$ and $\ell = 4$ in equation (i)

$$A = 3(4)^2$$

$$A = 3(16)$$

$$\boxed{A = 48 \text{ square units}}$$

(ii) To find ℓ ,

Put $k = 3$ and $A = 12$ in equation (i)

$$A = K\ell^2$$

$$12 = 3\ell^2$$

$$\frac{12}{3} = \ell^2$$

$$\ell^2 = 4$$

$$\sqrt{\ell^2} = \pm\sqrt{4}$$

$$\ell = \pm 2 \text{ units}$$

Since length is always taken positive so

$$\boxed{\ell = 2 \text{ units}}$$

Q. 2 The surface area S of the sphere varies directly as the square of radius r , and $S = 16\pi$ when $r = 2$. Find r when $S = 36\pi$.

Solution:

$$S \propto r^2 \quad \boxed{S = 16\pi, r = 2} \quad \boxed{r = ?, S = 36\pi}$$

$$S \propto r^2$$

$$S = kr^2 \dots\dots\dots (i)$$

To find k ,

Put $s = 16\pi$ and $r = 2$ in equation (i)

$$16\pi = k(2)^2$$

$$16\pi = k(4)$$

$$\frac{4\cancel{16}\pi}{\cancel{4}} = k$$

$$4\pi = k$$

$$\Rightarrow \boxed{k = 4\pi}$$

To find s ,

Put $k = 4\pi$ in equation (i)

$$S = kr^2$$

$$S = 4\pi r^2 \dots\dots\dots (ii)$$

To find r ,

Now put $S = 36\pi$ in equation (ii)

$$36\pi = 4\pi r^2$$

$$\frac{36\pi}{4\pi} = r^2$$

$$\frac{\cancel{9} \cancel{36} \cancel{\pi}}{\cancel{4} \cancel{\pi}} = r^2$$

$$r^2 = 9$$

$$r^2 = 9$$

Taking square root

$$\sqrt{r^2} = \pm \sqrt{9}$$

$$r = \pm 3$$

Since r is radius of sphere which is positive quantity so $\boxed{r = 3 \text{ units}}$

Q.3 In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S , and $F = 32 \text{ lb}$ when $S = 1.6 \text{ in}$. Find (i) S when $F = 50 \text{ lb}$ (ii) F when $S = 0.8 \text{ in}$.

Solution: $F \propto S$ $\boxed{F = 32 \text{ lb}, S = 1.6 \text{ inch}}$

$$\boxed{(i) S = ?, F = 50 \text{ lb}}$$

$$\boxed{(ii) F = ?, S = 0.8 \text{ inch}}$$

$$F \propto S$$

$$F = kS \dots\dots\dots (i)$$

To find k ,

put $F = 32$ and $s = 1.6$ in equation (i)

$$32 = k(1.6)$$

$$\frac{32}{1.6} = k$$

$$20 = k$$

$$k = 20$$

(i) To find s ,

Put $k = 20$ and $F = 50$ in equation. (i)

$$F = kS$$

$$50 = 20(S)$$

$$\frac{50}{20} = S$$

$$S = \frac{5}{2}$$

$$S = 2.5 \text{ inch}$$

(ii) To find F ,

Put $K = 20$ and $S = 0.8 \text{ inch}$ in equation (i)

$$F = kS$$

$$F = 20(0.8)$$

$$F = \cancel{20} \frac{8}{\cancel{10}_1}$$

$$F = 2 \times 8$$

$$F = 16 \text{ lb}$$

Q.4 The intensity I of the light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12 ft. from the source, Find the intensity at a point 8ft. from the source.

Solution: $I \propto \frac{1}{d^2}$ $I = 20 \text{ cp}$
 $d = 12 \text{ ft}$ $I = ?$
 $d = 8 \text{ ft}$

$$I \propto \frac{1}{d^2}$$

$$I = \frac{k}{d^2} \dots\dots\dots (i)$$

To find k ,

Put $I = 20$ and $d = 12 \text{ ft}$ in equation (i)

$$20 = \frac{k}{(12)^2}$$

$$20 = \frac{k}{144}$$

$$20 \times 144 = k$$

$$2880 = k$$

$$k = 2880$$

To find I ,

Now Put $k = 2880$ and $d = 8 \text{ ft}$ in equation (i)

$$I = \frac{k}{d^2}$$

$$I = \frac{2880}{(8)^2}$$

$$I = \frac{2880}{64}$$

$$I = 45 \text{ Candle power}$$

Q.5 The pressure P in a body of fluid, varies directly as the depth d . If the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 lb/sq. in, how deep must the fluid be to exert a pressure of 9 lb/sq.in?

Solution:

$$p \propto d$$

$$d = 5 \text{ ft}, p = 2.25 \text{ lb/sq. inch}$$

$$d = ?, p = 9 \text{ lb/sq. inch}$$

$$p \propto d$$

$$p = kd \dots\dots\dots (i)$$

To find k ,

Put $p = 2.25$ and $d = 5$ in equation (i)

$$2.25 = k(5)$$

$$\frac{2.25}{5} = k$$

$$\Rightarrow k = 0.45$$

To find d ,

Put $k = 0.45$ and $p = 9$ in equation (i)

$$p = kd$$

$$9 = 0.45(d)$$

$$\frac{9}{0.45} = d$$

$$d = \frac{900}{45} \Rightarrow \boxed{d = 20 \text{ ft}}$$

Q.6 Labour costs c varies jointly as the number of workers n and the average number of days d , if the cost of 800 workers for 13 days is Rs. 286000, then find the labour cost of 600 workers for 18 days.

Solution: $c \propto nd$

$$c = \text{Rs. } 286000$$

$$n = 800, d = 13$$

$$c = ?$$

$$n = 600, d = 18$$

$$c = knd \dots\dots\dots (i)$$

To find k ,

Put the values of c , n and d in equation (i)

$$286000 = k(800)(13)$$

$$\frac{286000}{800 \times 13} = k$$

$$27.5 = k \Rightarrow \boxed{k = 27.5}$$

To find C ,

Put $k = 27.5$, $n = 600$ and $d = 18$ in equation (i)

$$c = knd$$

$$c = 27.5 \times 600 \times 18$$

$$\boxed{c = \text{Rs. } 297000}$$

Q.7 The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length ℓ . A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons?

Solution:

$c \propto \frac{d^4}{\ell^2}$	$d = 6 \text{ inch}$	$\ell = ?$
	$\ell = 30 \text{ ft}$	$d = 4 \text{ inch}$
	$c = 63 \text{ tons}$	$c = 28 \text{ tons}$

$$c \propto \frac{d^4}{\ell^2}$$

$$c = \frac{kd^4}{\ell^2} \dots\dots\dots (i)$$

To find k,
Put the values of d , ℓ and c in equation (i)

$$63 = \frac{k(6)^4}{(30)^2}$$

$$63 = \frac{k(1296)}{900}$$

$$\frac{63 \times 900}{1296} = k$$

$$\frac{56700}{1296} = k$$

$$43.75 = k$$

$$k = 43.75$$

To find ℓ ,
Put $k = 43.75$, $d = 4$ and $c = 28$ in equation (i)

$$c = \frac{kd^4}{\ell^2}$$

$$28 = \frac{43.75 \times (4)^4}{\ell^2}$$

$$\ell^2 \times 28 = 43.75 \times 256$$

$$\ell^2 = \frac{11200}{28}$$

$$\ell^2 = 400$$

Taking square root of both side

$$\sqrt{\ell^2} = \pm \sqrt{400}$$

$$\ell = \pm 20$$

Since length is always positive so

$$\boxed{\ell = 20 \text{ ft}}$$

Q.8 The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power p of the motor. If 25 sec. are required for a 4-hp motor to lift 500 lb through 40 ft, what power is required to lift 800 lb, through 120 ft in 40 sec?

Solution:

$T \propto \frac{wd}{P}$

$T = 25 \text{ s}, \quad p = 4 \text{ hp}$	$p = ?, \quad w = 800 \text{ lb}$
$w = 500 \text{ lb}, d = 40 \text{ ft}$	$d = 120 \text{ ft}, T = 40 \text{ sec}$

$$T \propto \frac{wd}{p}$$

$$T = \frac{kwd}{p} \dots\dots\dots (i)$$

To find k,
Put the values of T , P , W and d in equation (i)

$$25 = \frac{k(500)(40)}{4}$$

$$\frac{25 \times 4}{500 \times 40} = k$$

$$\frac{100}{20000} = k$$

$$\frac{1}{200} = k \Rightarrow \boxed{k = \frac{1}{200}}$$

To find P,
Put, $k = \frac{1}{200}$, $w = 800 \text{ lb}$, $d = 120 \text{ ft}$,

$T = 40 \text{ s}$ in equation (i)

$$T = \frac{kwd}{p}$$

$$40 = \frac{1}{200} \times \frac{800 \times 120}{p}$$

$$40 = \frac{480}{p}$$

$$p = \frac{480}{40}$$

$$p = \frac{12}{1} = 12$$

$$\Rightarrow \boxed{p = 12 \text{ hp}}$$

Q.9 The kinetic energy (K.E) of a body varies jointly as the mass “m” of the body and the square of its velocity “v”. If the kinetic energy is 4320 ft/ℓ b when the mass is 45 ℓ b and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 ℓ b automobile travelling 44 ft/ sec.

Solution: $K.E \propto mv^2$

$K.E = 4320 \text{ ft} / \ell b$ $m = 45 \ell b$ $v = 24 \text{ ft} / s$
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$K.E = ?$ $m = 3000 \ell b$ $v = 44 \text{ ft} / \text{sec}$
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$$K.E \propto mv^2$$

$$K.E = kmv^2 \dots\dots\dots (i)$$

To find K,

Put K.E = 4320, m = 45 and v = 24 in equation (i)

$$4320 = k (45) (24)^2$$

$$4320 = k (45 \times 576)$$

$$4320 = k 25920$$

$$\frac{4320}{25920} = k$$

$k = \frac{432}{2592}$

To find K. E,

Put $k = \frac{432}{2592}$, m = 3000, v = 44 in equation (i)

$$K.E = Kmv^2$$

$$K.E = \frac{432}{2592} \times 3000 \times (44)^2$$

$$K.E = \frac{432}{2592} \times 3000 \times 1936$$

$$K.E = \frac{250905600}{2592}$$

$K.E = 96,8000 \text{ ft} / \ell b$
