

## MISCELLANEOUS EXERCISE - 3

### Q. 1 Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

- If  $u \propto v^2$ , then
  - $u \propto v^2$
  - $u = kv^2$
  - $uv^2 = k$
  - $uv^2 = 1$
- The third proportional of  $x^2$  and  $y^2$  is
  - $\frac{y^2}{x^2}$
  - $x^2 y^2$
  - $\frac{y^4}{x^2}$
  - $\frac{y^2}{x^4}$
- The fourth proportional w of  $x:y::v:w$  is.....
  - $\frac{xy}{v}$
  - $\frac{vy}{x}$
  - $xyv$
  - $\frac{x}{vy}$
- If  $a:b=x:y$ , then alternando property is
  - $\frac{a}{x} = \frac{b}{y}$
  - $\frac{a}{b} = \frac{x}{y}$
  - $\frac{a+b}{b} = \frac{x+y}{y}$
  - $\frac{a-b}{x} = \frac{x-y}{y}$
- In continued proportion  $a:b = b:c$ ,  $ac = b^2$ ,  $b$  is said to be \_\_\_ proportional.
  - third
  - fourth
  - means
  - none of these
- In continued proportion  $a:b = b:c$ ,  $c$  is said to be \_\_\_ proportional to  $a$  and  $b$ .
  - third
  - fourth
  - means
  - None of these
- If  $a : b = x : y$ , then invertendo property is
  - $\frac{a}{x} = \frac{b}{y}$
  - $\frac{a}{a-b} = \frac{x}{x-y}$
  - $\frac{a+b}{b} = \frac{x+y}{y}$
  - $\frac{b}{a} = \frac{y}{x}$
- In a ratio  $a:b$ ,  $a$  is called
  - relation
  - antecedent
  - consequent
  - None of these

9. If  $\frac{a}{b} = \frac{c}{d}$ , then componendo property is

- (a)  $\frac{a}{a+b} = \frac{c}{c+d}$  (b)  $\frac{a}{a-b} = \frac{c}{c-d}$   
 (c)  $\frac{ad}{bc}$  (d)  $\frac{a-b}{b} = \frac{c-d}{d}$

10. If  $\frac{24}{7} = \frac{6}{x}$ , then  $4x = \dots\dots\dots$

- (a) 7 (b)  $\frac{7}{4}$   
 (c) 4 (d)  $\frac{42}{24}$

11. In a ratio  $x: y$ ,  $y$  is called

- (a) relation (b) antecedent  
 (c) consequent (d) None of these

12. In a proportion  $a: b:: c: d$ ,  $a$  and  $d$  are called

- (a) means  
 (b) extremes  
 (c) fourth proportional  
 (d) None of these

13. In a proportion  $a:b::c: d$ ,  $b$  and  $c$  are called

- (a) means  
 (b) extremes  
 (c) fourth proportional  
 (d) None of these

14. Find  $x$  in proportion  $4:x::5:15$

- (a)  $\frac{75}{4}$  (b)  $\frac{4}{3}$   
 (c)  $\frac{3}{4}$  (d) 12

15. If  $y^2 \propto \frac{1}{x^3}$ , then

- (a)  $y^2 = \frac{k}{x^3}$  (b)  $y^2 = \frac{1}{x^3}$   
 (c)  $y^2 = x^2$  (d)  $y^2 = kx^3$

16. If  $w$  varies inversely as  $p^2$ , then  $k = \dots\dots$

- (a)  $\frac{w}{p^2}$  (b)  $wp^2$   
 (c)  $\frac{p^2}{w}$  (d)  $wp$

17. If  $6: m:: 9: 12$ , then  $m = \dots\dots$

- (a) 6 (b) 9  
 (c) 1 (d) 8

18. If  $x$  and  $y$  varies directly, then  $x = \dots\dots$

- (a)  $y$  (b)  $ky$   
 (c)  $\frac{k}{y}$  (d)  $k$

19. If  $\frac{u}{v} = \frac{v}{w} = k$ , then

- (a)  $u = wk^2$  (b)  $u = vk^2$   
 (c)  $u = w^2k$  (d)  $u = v^2k$

20. If  $\frac{5a}{3x} = \frac{15b}{y}$ , then  $ay = \dots\dots$

- (a)  $\frac{9bx}{y}$  (b)  $\frac{9y}{9b}$   
 (c)  $5ay = 45bx$  (d)  $9bx$

21. Fourth proportional of 5, 8, 15 is  $\dots\dots\dots$

- (a) 120 (b) 40  
 (c) 24 (d) 20

22. Newton's law of Gravitation is an example of

- (a) variation  
 (b) direct variation  
 (c) inverse variation  
 (d) joint variation

23. The relation between radius and circumference of a circle is an example of  $\dots\dots\dots$

- (a) variation  
 (b) direct variation  
 (c) inverse variation  
 (d) joint variation

24. If  $v$  varies directly as  $u^3$ , then  $u^3 = \dots\dots$

- (a)  $vk$  (b)  $\frac{k}{v}$   
 (c)  $\frac{v}{k}$  (d)  $vk^3$

25. The simplest form of the ratio

$$\frac{(x+y)(x^2+xy+y^2)}{x^3-y^3} \text{ is } \dots\dots\dots$$

- (a)  $\frac{x-y}{x+y}$  (b)  $\frac{x+y}{x-y}$   
 (c) 1 (d) 2



**Q.2:** Write short answer of the following questions:

(i) Define ratio and give one example.

**Answer:** A relation between two quantities of the same kind is called ratio and ratio has no unit. If 'a' and 'b' are two quantities of the same kind and b is not zero, then their ratio is defined as a : b or  $\frac{a}{b}$  or  $a \div b$ .

**Example:** If a hockey team wins 4 matches and losses 5, then ratio of the games won to the games lost is 4 : 5 or in fraction  $\frac{4}{5}$ .

(ii) Define proportion.

**Proportion:**

A proportion is a statement, which is expressed as an equivalence of two ratios.

If two ratios a : b and c : d are equal, then we can write a : b = c : d

Symbolically the proportion of a, b, c and d is written as

$$a : b :: c : d$$

(iii) Define direct variation.

**Direct variation:**

If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity in the same ratio, then this variation is called direct variation.

If a quantity y varies directly with regard to a quantity x, we say that y is directly proportional to x and is written as

$$y \propto x \text{ or } y = kx \text{ where } k \neq 0.$$

(iv) Define inverse variation.

**Inverse Variation:**

If two quantities are related in such a way that when one quantity increases, the other decreases in the same ratio is called inverse variation.

If a quantity y varies inversely with regard to quantity x, we say that y is inversely proportional to x or y varies inversely as x and is written as

$$y \propto \frac{1}{x} \text{ or } y = \frac{k}{x}$$

i.e.  $xy = k$ , where  $k \neq 0$  is the constant of variation.

(v) State theorem of componendo–dividendo.

**Theorem of componendo–dividendo**

If  $a : b = c : d$ , then componendo–dividendo theorem is written as:

$$(i) a + b : a - b = c + d : c - d$$

$$(ii) a - b : a + b = c - d : c + d$$

(vi) Find x, if 6:x::3:5.

**Solution:**

$$6 : x :: 3 : 5$$

Product of Means = Product of Extremes:

$$3(x) = 6 \times 5$$

$$3x = 30$$

$$x = \frac{30}{3}$$

$$\boxed{x = 10}$$

(vii) If x and  $y^2$  varies directly, and  $x = 27$  when  $y = 4$ . Find the value of y when  $x=3$ .

**Solution:**

$$\boxed{x \propto y^2} \quad \boxed{x = 27} \quad \boxed{y = ?}$$

$$\boxed{y = 4} \quad \boxed{x = 3}$$

$$y^2 \propto x$$

$$y^2 = kx \dots\dots\dots(i)$$

**To find K,**

Put  $x = 27$  and  $y = 4$  in equation (i)

$$(4)^2 = k(27)$$

$$\frac{16}{27} = k \Rightarrow \boxed{k = \frac{16}{27}}$$

**To find y,**

Put  $k = \frac{16}{27}$  and  $x = 3$  in equation (i)

$$y^2 = kx$$

$$y^2 = \frac{16}{27} \times 3$$

$$y^2 = \frac{16}{9}$$

$$\sqrt{y^2} = \pm \sqrt{\frac{16}{9}} \Rightarrow \boxed{y = \pm \frac{4}{3}}$$



(viii) If  $u$  and  $v$  varies inversely, and  $u = 8$ , when  $v = 3$ , find  $v$  when  $u = 12$ .

Solution:

$$\boxed{v \propto \frac{1}{u}} \quad \boxed{u = 8} \quad \boxed{v = ?}$$

$$\boxed{v = 3} \quad \boxed{u = 12}$$

$$v \propto \frac{1}{u}$$

$$v = \frac{k}{u} \dots\dots\dots(i)$$

To find  $k$ ,

Put  $u = 8$  and  $v = 3$  in equation (i)

$$3 = \frac{k}{8}$$

$$3 \times 8 = k$$

$$24 = k$$

$$\Rightarrow \boxed{k = 24}$$

To find  $y$ ,

Put  $k = 24$  and  $u = 12$  in equation (i)

$$v = \frac{k}{u}$$

$$v = \frac{24}{12}$$

$$\boxed{v = 2}$$

(ix) Find a fourth proportional to 8, 7, 6.

Solution:

Let 4<sup>th</sup> proportional is  $x$ .

then 8, 7, 6,  $x$

By Proportion

$$8 : 7 :: 6 : x$$

Product of Extremes = Product of Means:

$$8(x) = 7 \times 6$$

$$x = \frac{42}{8}$$

$$\boxed{x = \frac{21}{4}}$$

(x) Find a mean proportional to 16 and 49.

Solution:

Let mean proportional is  $m$ ,

Then 16,  $m$ , 49

By proportion

$$16 : m :: m : 49$$

Product of Means = Product of Extremes:

$$m \cdot m = 16 \times 49$$

$$m^2 = 4^2 \times 7^2$$

$$m^2 = (4 \times 7)^2$$

$$m^2 = (28)^2$$

Taking square root

$$\sqrt{m^2} = \pm \sqrt{(28)^2}$$

$$\boxed{m = \pm 28}$$

(xi) Find third proportional to 28 and 4.

Solution:

Let 3<sup>rd</sup> proportional is  $x$

Then 28, 4,  $x$

By Proportion

$$28 : 4 :: 4 : x$$

Product of Extremes = Product of Means.

$$28(x) = 4 \times 4$$

$$x = \frac{16}{28}$$

$$\boxed{x = \frac{4}{7}}$$

(xii) If  $y \propto \frac{x^2}{z}$  and  $y = 28$  when  $x = 7$ ,  $z = 2$ ,

then find  $y$ .

Solution:

$$\boxed{x \propto \frac{x^2}{z}} \quad \boxed{y = 28} \quad \boxed{y = ?}$$

$$\boxed{x = 7} \quad \boxed{z = 2}$$

$$y \propto \frac{x^2}{z}$$

$$y = \frac{kx^2}{z} \dots\dots\dots(i)$$

To find  $k$ ,

Put  $y = 28$ ,  $x = 7$  and  $z = 2$  in equation (i)

$$28 = \frac{k(7)^2}{2}$$

$$28 \times 2 = k(49)$$

$$\frac{56}{49} = k$$

$$k = \frac{56}{49} \cdot \frac{8}{7}$$

$$\boxed{k = \frac{8}{7}}$$

To find y,

Put  $k = \frac{8}{7}$  in equation (i)

$$y = k \frac{x^2}{z}$$

$$y = \frac{8}{7} \frac{x^2}{z}$$

(xiii) If  $z \propto xy$  and  $z = 36$ , when  $x = 2$ ,  $y = 3$ , then find z.

Solution:

$$\boxed{z \propto xy} \quad \boxed{z = 36, x = 2, y = 3} \quad \boxed{z = ?}$$

$$z \propto xy$$

$$z = kxy \dots \dots \dots (i)$$

To find k,

put  $z = 36$ ;  $x = 2$  and  $y = 3$  in equation (i)

$$36 = k (2) (3)$$

$$36 = k (6)$$

$$\frac{36}{6} = k$$

$$6 = k$$

$$\Rightarrow \boxed{k = 6}$$

To find z,

Put  $k = 6$  in equation (i)

$$z = k xy$$

$$\boxed{z = 6xy}$$

(xiv) If  $w \propto \frac{1}{v^2}$  and  $w = 2$  when  $v = 3$  then

find w.

Solution:

$$\boxed{w \propto \frac{1}{v^2}} \quad \boxed{\frac{w}{v} = \frac{2}{3}} \quad \boxed{w = ?}$$

$$w \propto \frac{1}{v^2}$$

$$w = \frac{k}{v^2} \dots \dots \dots (i)$$

To find K,

Put  $w = 2$  and  $v = 3$  in equation (i)

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$2 \times 9 = k$$

$$18 = k$$

$$\Rightarrow \boxed{k = 18}$$

To find w,

Put  $k = 18$  in equation (i)

$$w = \frac{k}{v^2}$$

$$w = \frac{18}{v^2}$$

Fill in the blanks

- The simplest form of the ratio  $\frac{(x+y)(x^2+xy+y^2)}{x^3-y^3}$  is \_\_\_\_\_.
- In a ratio  $x : y$ ; x is called \_\_\_\_\_.
- In a ratio  $a : b$ , b is called \_\_\_\_\_.
- In a proportion  $a : b :: x : y$ , a and y are called \_\_\_\_\_.
- In a proportion  $p : q :: m : n$ ; q and m are called \_\_\_\_\_.
- In proportion  $7 : 4 :: p : 8$ ,  $p =$  \_\_\_\_\_.
- If  $6 : m :: 9 : 12$ , then  $m =$  \_\_\_\_\_.
- If x and y varies directly, the  $x =$  \_\_\_\_\_.
- If v varies directly as  $u^3$ , then  $u^3 =$  \_\_\_\_\_.
- If w varies inversely as  $p^2$ , then  $k =$  \_\_\_\_\_.
- A third proportional of 12 and 4, is \_\_\_\_\_.
- The fourth proportional of 15, 6, 5 is \_\_\_\_\_.
- The mean proportional of  $4m^2n^4$  and  $p^6$  is \_\_\_\_\_.
- The continued proportion of 4, m and 9 is \_\_\_\_\_.

ANSWER KEY

1	$\frac{x+y}{x-y}$	2	Antecedent	3	Consequent
4	Extremes	5	Means	6	$P = 14$
7	$m = 8$	8	ky	9	$\frac{v}{k}$
10	$p^2w$	11	$\frac{4}{3}$	12	2
13	$\pm 2mn^2 p^3$	14	$m = \pm 6$		