EXERCISE 1.1

Q.1 Write the following quadratic equations in the Standard form and point out pure quadratic equations.

(i) $(x+7)(x-3)=-7$

Solution: $(x+7)(x-3)=-7$

$$x^2-3x+7x-21=-7$$
$$x^2+4x-21+7=0$$
$$x^2+4x-14=0$$

The standard form of quadratic equation is:

$$x^2+4x-14=0$$

(ii) $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

Solution: $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

$$\frac{7(x^2+4)-3x}{21}=1$$
$$7x^2+28-3x=21$$
$$7x^2-3x+28-21=0$$
$$7x^2-3x+7=0$$

The standard form of quadratic equation is:

$$7x^2-3x+7=0$$

(iii) $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

Solution: $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

$$\frac{x^2+(x+1)^2}{x(x+1)}=6$$
$$x^2+x^2+1+2x=6x(x+1)$$
$$2x^2+2x+1=6x^2+6x$$
$$0=6x^2+6x-2x^2-2x-1$$
$$0=4x^2+4x-1$$
$$\Rightarrow \quad 4x^2+4x-1=0$$

The standard form of quadratic equation is:

$$4x^2+4x-1=0$$

(iv) $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

Solution: $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

$$\frac{(x+4)x-(x-2)^2+4x(x-2)}{(x-2)(x)}=0$$
$$x^2+4x-[x^2+2^2-2(x)(2)]+4x^2-8x=0$$
$$\cancel{x^2}+4x-\cancel{x^2}-4+4x+4x^2-8x=0$$
$$4x^2+4x+4x-8x-4=0$$
$$4x^2+\cancel{8x}-\cancel{8x}-4=0$$
$$4x^2-4=0$$
$$4(x^2-1)=0$$
$$\therefore \quad x^2-1=0 \quad (\because 4\neq 0)$$

So, $x^2-1=0$ is Pure Quadratic Equation

(v) $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

Solution: $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$
$$x^2+3x-(x^2-5x+4x-20)=1x(x+4)$$
$$x^2+3x-(x^2-1x-20)=x^2+4x$$
$$\cancel{x^2}+3x-\cancel{x^2}+x+20=x^2+4x$$
$$3x+x+20=x^2+4x$$
$$4x+20=x^2+4x$$
$$\Rightarrow \quad x^2+\cancel{4x}-\cancel{4x}-20=0$$
$$x^2-20=0$$
$$x^2+0x-20=0$$
$$\text{As, } b=0$$

So, $x^2-20=0$ is Pure Quadratic Equation

(vi) $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+3}=\dfrac{25}{12}$

Solution: $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+3}=\dfrac{25}{12}$

$$\frac{(x+1)(x+3)+(x+2)(x+2)}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{(x^2+3x+1x+3)+(x^2+2x+2x+4)}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{(x^2+4x+3)+(x^2+4x+4)}{x^2+3x+2x+6}=\frac{25}{12}$$

$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$

$$12(2x^2+8x+7)=25(x^2+5x+6)$$

$$24x^2+96x+84=25x^2+125x+150$$

$$25x^2-24x^2+125x-96x+150-84=0$$

$$x^2+29x+66=0$$

The standard form of quadratic equation is:

$$x^2+29x+66=0$$

Q.2 Solve by Factorization:

(i) $x^2-x-20=0$

Solution:

$$x^2-x-20=0$$

$$x^2-5x+4x-20=0$$

$$x(x-5)+4(x-5)=0$$

$$(x-5)(x+4)=0$$

$$x-5=0 \quad \text{or} \quad x+4=0$$

$$x=5 \quad \text{or} \quad x=-4$$

Solution set is $\{-4, 5\}$

(ii) $3y^2=y(y-5)$

Solution: $3y^2=y(y-5)$

$$3y^2=y^2-5y$$

$$3y^2-y^2+5y=0$$

$$2y^2+5y=0$$

$$y(2y+5)=0$$

Either $y=0$ or $2y+5=0$

$$y=0 \quad \text{or} \quad 2y=-5$$

$$y=0 \quad \text{or} \quad y=\frac{-5}{2}$$

Solution is $\left\{0, \dfrac{-5}{2}\right\}$

(iii) $4-32x=17x^2$

Solution: $4-32x=17x^2$

$$17x^2+32x-4=0$$

$$17x^2+34x-2x-4=0$$

$$17x(x+2)-2(x+2)=0$$

$$(x+2)(17x-2)=0$$

Either $x+2=0$ or $17x-2=0$

$$x=-2 \quad \text{or} \quad 17x=2$$

$$x=-2 \quad \text{or} \quad x=\frac{2}{17}$$

Solution set is $\left\{-2, \dfrac{2}{17}\right\}$

(iv) $x^2-11x=152$

Solution: $x^2-11x=152$

$$x^2-11x-152=0$$

$$x^2-19x+8x-152=0$$

$$x(x-19)+8(x-19)=0$$

$$(x-19)(x+8)=0$$

$$x-19=0 \quad \text{or} \quad x+8=0$$

$$x=19 \quad \text{or} \quad x=-8$$

Solution set is $\{-8, 19\}$

(v) $\dfrac{x+1}{x}+\dfrac{x}{x+1}=\dfrac{25}{12}$

Solution: $\dfrac{x+1}{x}+\dfrac{x}{x+1}=\dfrac{25}{12}$

$$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$$

$$\frac{(x)^2+(1)^2+2(x)(1)+x^2}{x^2+x}=\frac{25}{12}$$

$$\frac{x^2+1+2x+x^2}{x^2+x}=\frac{25}{12}$$

$12(2x^2+2x+1)=25\,(x^2+x)$

$24x^2+24x+12=25x^2+25x$

$0=25x^2+25x-24x^2-24x-12$

$0=x^2+x-12$

$\Rightarrow \quad x^2+x-12=0$

$x^2+4x-3x-12=0$

$x(x+4)-3\,(x+4)=0$

$(x+4)(x-3)=0$

Either

$x+4=0 \quad$ or $\quad x-3=0$

$\Rightarrow \quad x=-4 \quad$ or $\quad x=3$

Solution set is $\{3,-4\}$

(vi) $\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$

Solution: $\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$

$$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$$

$$\frac{2}{x-9}=\frac{\cancel{x}-4-\cancel{x}+3}{x^2-4x-3x+12}$$

$$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$$

$2(x^2-7x+12)=-1(x-9)$

$2x^2-14x+24=-1x+9$

$2x^2-14x+x+24-9=0$

$2x^2-13x+15=0$

$2x^2-10x-3x+15=0$

$2x(x-5)-3(x-5)=0$

$(x-5)(2x-3)=0$

Either $\quad x-5=0 \quad$ or $\quad 2x-3=0$

$\quad\quad x=5 \quad$ or $\quad 2x=3$

$\quad\quad x=5 \quad$ or $\quad x=\dfrac{3}{2}$

Solution set is $\left\{5,\dfrac{3}{2}\right\}$

Q.3 Solve the following equations by completing square.

(i) $\mathbf{7x^2+2x-1=0}$

Solution: $7x^2+2x-1=0$

Dividing each term of the equation by 7

$$\frac{\cancel{7}}{\cancel{7}}x^2+\frac{2}{7}x-\frac{1}{7}=0$$

$$x^2+\frac{2}{\cancel{2}}(x)\left(\frac{\cancel{2}}{7}\right)^1-\frac{1}{7}=0$$

$$(x)^2+2(x)\left(\frac{1}{7}\right)=\frac{1}{7}$$

Adding $\left(\frac{1}{7}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$

$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{7+1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Taking square root of both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{8}{49}}$$

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{4\times2}{49}}$$

$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$

$$x=\frac{-1}{7}\pm\frac{2\sqrt{2}}{7}$$

$$\therefore\quad x=\frac{-1\pm2\sqrt{2}}{7}$$

Solution set is $\left\{\frac{-1\pm2\sqrt{2}}{7}\right\}$

(ii) $\mathbf{ax^2+4x-a=0}$, $a\neq0$

Solution: $ax^2+4x-a=0$, $a\neq0$

Dividing each term of the equation by 'a'

$$\frac{ax^2}{a}+4\frac{x}{a}-\frac{a}{a}=\frac{0}{a}$$

$$x^2+4\frac{x}{a}-1=0$$

$$x^2+\frac{2}{\cancel{2}}(x)\left(\frac{\cancel{4}^{2}}{a}\right)=1$$

$$(x)^2+2(x)\left(\frac{2}{a}\right)=1$$

Adding $\left(\frac{2}{a}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$$

Taking square root of both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2}=\pm\sqrt{\frac{a^2+4}{a^2}}$$

$$x+\frac{2}{a}=\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2}{a}\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2\pm\sqrt{a^2+4}}{a}$$

Solution set is $\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

(iii) $\mathbf{11x^2-34x+3=0}$

Solution: $11x^2-34x+3=0$

Dividing each term of the equation by 11

$$\frac{\cancel{11}}{\cancel{11}}x^2-\frac{34}{11}x+\frac{3}{11}=0$$

$$x^2-\frac{34}{11}x=-\frac{3}{11}$$

$$(x)^2-\frac{2}{\cancel{2}}(x)\left(\frac{\cancel{34}^{17}}{11}\right)=-\frac{3}{11}$$

$$(x)^2-2(x)\left(\frac{17}{11}\right)=-\frac{3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides we get

$$(x)^2 - 2(x)\left(\frac{17}{11}\right) + \left(\frac{17}{11}\right)^2 = -\frac{3}{11} + \left(\frac{17}{11}\right)^2$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-3}{11} + \frac{289}{121}$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-33 + 289}{121}$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{256}{121}$$

Taking square root

$$\sqrt{\left(x - \frac{17}{11}\right)^2} = \pm\sqrt{\frac{256}{121}}$$

$$x - \frac{17}{11} = \pm\frac{16}{11}$$

$$x = \frac{17}{11} \pm \frac{16}{11}$$

$$x = \frac{17 \pm 16}{11}$$

$\Rightarrow$ $x = \frac{17-16}{11}$ or $x = \frac{17+16}{11}$

$x = \frac{1}{11}$ or $x = \frac{33}{11}$

$x = 3$

Solution set is $\left\{3, \frac{1}{11}\right\}$

(iv) $\ell x^2 + mx + n = 0$, $\ell \neq 0$

Solution: $\ell x^2 + mx + n = 0$, $\ell \neq 0$

Dividing each term of the equation by ℓ

$$\frac{\ell x^2}{\ell} + (x)\left(\frac{m}{\ell}\right)x = -\frac{n}{\ell}$$

$$x^2 + \frac{2}{2}(x)\left(\frac{m}{\ell}\right) = -\frac{n}{\ell}$$

$$x^2 + 2(x)\left(\frac{m}{2\ell}\right) = -\frac{n}{\ell}$$

Adding $\left(\frac{m}{2\ell}\right)^2$ on both sides

$$(x)^2 + 2(x)\left(\frac{m}{2\ell}\right) + \left(\frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \left(\frac{m}{2\ell}\right)^2$$

$$\left(x + \frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \frac{m^2}{4\ell^2}$$

$$\left(x + \frac{m}{2\ell}\right)^2 = \frac{-4\ell n + m^2}{4\ell^2}$$

Taking square root

$$\sqrt{\left(x + \frac{m}{2\ell}\right)^2} = \pm\sqrt{\frac{m^2 - 4\ell n}{4\ell^2}}$$

$$x + \frac{m}{2\ell} = \pm\frac{\sqrt{m^2 - 4\ell n}}{2\ell}$$

$$x = -\frac{m}{2\ell} \pm \frac{\sqrt{m^2 - 4\ell n}}{2\ell}$$

$$x = \frac{-m \pm \sqrt{m^2 - 4\ell n}}{2\ell}$$

Solution set is $\left\{\frac{-m \pm \sqrt{m^2 - 4\ell n}}{2\ell}\right\}$

(v) $3x^2 + 7x = 0$

Solution: $3x^2 + 7x = 0$

Dividing each term of equation by 3

$$\frac{\cancel{3}x^2}{\cancel{3}} + \frac{7}{3}x = 0$$

$$x^2 + \frac{2}{2}(x)\left(\frac{7}{3}\right) = 0$$

$$(x)^2 + 2(x)\left(\frac{7}{6}\right) = 0$$

Adding $\left(\frac{7}{6}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\frac{49}{36}$$

Taking square root

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\frac{49}{36}}$$

$$\left(x+\frac{7}{6}\right)=\pm\frac{7}{6}$$

$$x=\frac{-7}{6}\pm\frac{7}{6}$$

$$x=\frac{-7\pm7}{6}$$

$$\Rightarrow \quad x=\frac{-7-7}{6} \quad \text{or} \quad x=\frac{-7+7}{6}$$

$$x=\frac{-14}{6} \quad \text{or} \quad x=\frac{0}{6}$$

$$x=\frac{-7}{3} \quad \text{or} \quad x=0$$

Solution set is $\left\{0, \frac{-7}{3}\right\}$

(vi) $x^2-2x-195=0$

Solution: $x^2-2x-195=0$

$$x^2-2x=195$$

$$x^2-\frac{2}{2}(x)(1)=195$$

$$(x)^2-2(x)(1)=195$$

Adding $(1)^2$ on both sides

$$(x)^2-2(x)(1)+(1)^2=195+(1)^2$$

$$(x-1)^2=195+1$$

$$(x-1)^2=196$$

Taking square root

$$\sqrt{(x-1)^2}=\pm\sqrt{196}$$

$$x-1=\pm14$$

$$x=1\pm14$$

$$\Rightarrow \quad x=1-14 \quad \text{or} \quad x=1+14$$

$$x=-13 \quad \text{or} \quad x=15$$

Solution set is $\{-13, 15\}$

(vii) $-x^2+\frac{15}{2}=\frac{7}{2}x$

Solution: $-x^2+\frac{15}{2}=\frac{7}{2}x$

$$\frac{15}{2}=x^2+\frac{7}{2}x$$

$$\Rightarrow \quad x^2+\frac{7}{2}x=\frac{15}{2}$$

$$x^2+\frac{2}{2}(x)\left(\frac{7}{2}\right)=\frac{15}{2}$$

$$(x)^2+2(x)\left(\frac{7}{4}\right)=\frac{15}{2}$$

Adding $\left(\frac{7}{4}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$$

$$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$$

Taking Square Root

$$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$$

$$x+\frac{7}{4}=\pm\frac{13}{4}$$

$$x=\frac{-7}{4}\pm\frac{13}{4}$$

$$x = \frac{-7 \pm 13}{4}$$

$\Rightarrow$ $x = \frac{-7-13}{4}$ or $x = \frac{-7+13}{4}$

$x = \frac{-20}{4}$ or $x = \frac{6}{4}$

$x = -5$ or $x = \frac{3}{2}$

Solution set is $\left\{-5, \frac{3}{2}\right\}$

(viii) $\mathbf{x^2 + 17x + \frac{33}{4} = 0}$

Solution: $x^2 + 17x + \frac{33}{4} = 0$

$$x^2 + 17x = -\frac{33}{4}$$

$$(x)^2 + 2(x)\left(\frac{17}{2}\right) = -\frac{33}{4}$$

Adding $\left(\frac{17}{2}\right)^2$ on both sides

$$(x)^2 + 2(x)\left(\frac{17}{2}\right) + \left(\frac{17}{2}\right)^2 = \frac{-33}{4} + \left(\frac{17}{2}\right)^2$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33}{4} + \frac{289}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33+289}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{256}{4}$$

Taking square root

$$\sqrt{\left(x + \frac{17}{2}\right)^2} = \pm\sqrt{\frac{256}{4}}$$

$$x + \frac{17}{2} = \pm\frac{16}{2}$$

$$x = \frac{-17}{2} \pm \frac{16}{2}$$

$$x = \frac{-17 \pm 16}{2}$$

$\Rightarrow$ $x = \frac{-17-16}{2}$ or $x = \frac{-17+16}{2}$

$x = \frac{-33}{2}$ or $x = \frac{-1}{2}$

Solution set is $\left\{-\frac{1}{2}, \frac{-33}{2}\right\}$

(ix) $\mathbf{4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}}$

Solution: $4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$

$$4 = \frac{3x^2+5}{3x+1} + \frac{8}{3x+1}$$

$$4 = \frac{3x^2+5+8}{3x+1}$$

$$4(3x+1) = 3x^2 + 13$$

$$12x + 4 = 3x^2 + 13$$

$$3x^2 + 13 - 12x - 4 = 0$$

$$3x^2 - 12x + 9 = 0$$

Dividing each term of the equation by 3

$$\frac{\cancel{3}}{\cancel{3}}x^2 - \frac{\cancel{12}^{4}x}{\cancel{3}} + \frac{\cancel{9}^{3}}{\cancel{3}} = 0$$

$$x^2 - 4x + 3 = 0$$

$$x^2 - 4x = -3$$

$$(x)^2 - 2(x)(2) = -3$$

Adding $(2)^2$ on both sides

$$(x)^2 - 2(x)(2) + (2)^2 = -3 + (2)^2$$

$$(x-2)^2 = -3 + 4$$

$$(x-2)^2 = 1$$

Taking square root

$$\sqrt{(x-2)^2} = \pm\sqrt{1}$$

$$x - 2 = \pm 1$$

$\Rightarrow$ $x - 2 = +1$ or $x - 2 = -1$

$x = 2 + 1$ $\quad$ $x = 2 - 1$

$x = 3$ or $x = 1$

Solution set is $\{1, 3\}$

(x) $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution: $7(x+2a)^2+3a^2=5a(7x+23a)$

$$7(x^2+4a^2+4ax)+3a^2=35ax+115a^2$$
$$7x^2+28a^2+28ax+3a^2-35ax-115a^2=0$$
$$7x^2-35ax+28ax+28a^2+3a^2-115a^2=0$$
$$7x^2-7ax-84a^2=0$$

Dividing each term of the equation by 7 we get

$$\frac{7x^2}{7}-\frac{7ax}{7}=\frac{12\,84a^2}{7}$$
$$x^2-ax=12a^2$$
$$(x)^2-2(x)\left(\frac{a}{2}\right)=12a^2$$

Adding $\left(\frac{a}{2}\right)^2$ an both sides

$$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$
$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root

$$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$$
$$x-\frac{a}{2}=\pm\frac{7a}{2}$$
$$x=\frac{a}{2}\pm\frac{7a}{2}$$
$$x=\frac{a\pm7a}{2}$$

$\Rightarrow$ $x=\frac{a+7a}{2}$ or $x=\frac{a-7a}{2}$

$x=\frac{8a}{2}$ or $x=\frac{-6a}{2}$

$x=4a$ or $x=-3a$

Solution set is $\{-3a, 4a\}$