EXERCISE 1.2

Q.1 Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

Solution: $2 - x^2 = 7x$

$1x^2 + 7x - 2 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 1,\ b = 7,\ c = -2$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$\therefore \quad x = \frac{-7 \pm \sqrt{57}}{2}$$

Solution set is $\left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$

(ii) $5x^2 + 8x + 1 = 0$

Solution: $5x^2 + 8x + 1 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 5$, $b = 8$, $c = 1$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm \sqrt{4 \times 11}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{\not{2}\left(-4 \pm \sqrt{11}\right)}{\not{10}\ 5}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

Solution: $\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}\ x^2 + 1x - 4\sqrt{3} = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = \sqrt{3}$, $b = 1$, $c = -4\sqrt{3}$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4\left(\sqrt{3}\right)\left(-4\sqrt{3}\right)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16\left(\sqrt{3}\right)^2}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(3)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{-1 - 7}{2\sqrt{3}} \quad \text{or} \quad x = \frac{-1 + 7}{2\sqrt{3}}$$

$$x = \frac{-\not{8}^{\,4}}{\not{2}\sqrt{3}} \quad \text{or} \quad x = \frac{\not{6}^{\,3}}{\not{2}\sqrt{3}}$$

$$x = \frac{-4}{\sqrt{3}} \quad \text{or} \quad x = \frac{3}{\sqrt{3}}$$

$$x = \frac{-4}{\sqrt{3}} \quad \text{or} \quad x = \sqrt{3} \qquad \left(\because \frac{a}{\sqrt{a}} = \sqrt{a}\right)$$

Solution set is $\left\{\sqrt{3}, -\frac{4}{\sqrt{3}}\right\}$

(iv) $\mathbf{4x^2 - 14 = 3}$

Solution: $4x^2 - 14 = 3x$

$4x^2 - 3x - 14 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 4, b = -3, c = -14$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

Solution set is $\left\{\frac{3 \pm \sqrt{233}}{8}\right\}$

(v) $\mathbf{6x^2 - 3 - 7x = 0}$

Solution: $\mathbf{6x^2 - 3 - 7x = 0}$

$6x^2 - 7x - 3 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 6$, $b = -7$, $c = -3$

Using Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$$\Rightarrow \quad x = \frac{7 - 11}{12} \quad \text{or} \quad x = \frac{7 + 11}{12}$$

$$x = \frac{-4}{12} \quad \text{or} \quad x = \frac{18}{12}$$

$$x = \frac{-1}{3} \quad \text{or} \quad x = \frac{3}{2}$$

Solution set is $\left\{\frac{-1}{3}, \frac{3}{2}\right\}$

(vi) $\mathbf{3x^2 + 8x + 2 = 0}$

Solution: $3x^2 + 8x + 2 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 3$, $b = 8$, $c = 2$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$$

$$x = \frac{-8 \pm \sqrt{40}}{6}$$

$$x = \frac{-8 \pm \sqrt{4 \times 10}}{6}$$

$$x = \frac{-8 \pm 2\sqrt{10}}{6}$$

$$x = \frac{\cancel{2}\left(-4 \pm \sqrt{10}\right)}{\cancel{6}3}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6}-\frac{4}{x-5}=1$

Solution: $\frac{3}{x-6}-\frac{4}{x-5}=1$

$$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$$

$$3(x-5)-4(x-6)=(x-6)(x-5)$$

$$3x-15-4x+24=x^2-5x-6x+30$$

$$-1x+9=x^2-11x+30$$

$$x^2-11x+1x+30-9=0$$

$$1x^2-10x+21=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=1$, $b=-10$, $c=21$

Using formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$$

$$x=\frac{10\pm\sqrt{100-84}}{2}$$

$$x=\frac{10\pm\sqrt{16}}{2}$$

$$x=\frac{10\pm4}{2}$$

$$x=\frac{\cancel{2}(5\pm2)}{\cancel{2}}$$

$\Rightarrow$ $x=5+2$ or $x=5-2$

$x=7$ or $x=3$

Solution set is $\{3, 7\}$

(viii) $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

Solution: $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

$$\frac{(x+2)2x-(4-x)(x-1)}{(x-1)(2x)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-4-x^2+x)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{2x^2+4x-5x+4+x^2}{2x^2-2x}=\frac{7}{3}$$

$$\frac{3x^2-x+4}{2x^2-2x}=\frac{7}{3}$$

$$3(3x^2-x+4)=7(2x^2-2x)$$

$$9x^2-3x+12=14x^2-14x$$

$\Rightarrow$ $14x^2-9x^2-14x+3x-12=0$

$$5x^2-11x-12=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=5$, $b=-11$, $c=-12$

Using formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$

$$x=\frac{11\pm\sqrt{121+240}}{10}$$

$$x=\frac{11\pm\sqrt{361}}{10}$$

$$x=\frac{11\pm19}{10}$$

$\Rightarrow$ $x=\frac{11-19}{10}$ or $x=\frac{11+19}{10}$

$x=\frac{-8}{10}$ or $x=\frac{30}{10}$

$x=-\frac{4}{5}$ or $x=3$

Solution set is $\left\{3, \frac{-4}{5}\right\}$

(ix) $\dfrac{a}{x-b}+\dfrac{b}{x-a}=2$

Solution: $\dfrac{a}{x-b}+\dfrac{b}{x-a}=2$

$$\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$$

$$\frac{ax-a^2+bx-b^2}{x^2-ax-bx+ab}=2$$

$$ax-a^2+bx-b^2=2(x^2-ax-bx+ab)$$

$$ax-a^2+bx-b^2=2x^2-2ax-2bx+2ab$$

$$2x^2-2ax-2bx+2ab-ax+a^2-bx+b^2=0$$

$$2x^2-2ax-ax-2bx-bx+2ab+a^2+b^2=0$$

$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$

$$2x^2-(3a+3b)x+(a^2+b^2+2ab)=0$$

$$2x^2-(3a+3b)x+(a+b)^2=0$$

As $Ax^2+Bx+C=0$

$\Rightarrow$ $A=2$, $B=-(3a+3b)$, $C=(a+b)^2$

Using Formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-[-(3a+3b)]\pm\sqrt{[-(3a+3b)]^2-4(2)(a+b)^2}}{2(2)}$$

$$x=\frac{+3(a+b)\pm\sqrt{[-3(a+b)]^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{\left[(-3)^2(a+b)\right]^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm(a+b)}{4}$$

$x=\dfrac{3(a+b)-(a+b)}{4}$ or $x=\dfrac{3(a+b)+(a+b)}{4}$

$x=\dfrac{2(a+b)}{4}$ or $x=\dfrac{4(a+b)}{4}$

$x=\dfrac{a+b}{2}$ or $x=a+b$

$x=\dfrac{1}{2}(a+b)$ or $x=a+b$

Solution set is $\left\{(a+b),\ \dfrac{1}{2}(a+b)\right\}$

(x) $-(\ell+m)-\ell x^2+(2\ell+m)x=0$, $\ell\neq 0$

Solution: $-(\ell+m)-\ell x^2+(2\ell+m)x=0$, $\ell\neq 0$

$\Rightarrow$ $(\ell+m)+\ell x^2-(2\ell+m)x=0$ 01(032)

$\Rightarrow$ $\ell x^2-(2\ell+m)x+(\ell+m)=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=\ell$, $b=-(2\ell+m)$, $c=\ell+m$

Using Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-[-(2\ell+m)]\pm\sqrt{[-(2\ell+m)]^2-4(\ell)(\ell+m)}}{2(\ell)}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(-1)^2(2\ell+m)^2-4\ell(\ell+m)}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(2\ell)^2+(m)^2+2(2\ell)(m)-4\ell^2-4\ell m}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{\cancel{4\ell^2}+m^2+\cancel{4\ell m}-\cancel{4\ell^2}-\cancel{4\ell m}}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{m^2}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm m}{2\ell}$$

$x=\dfrac{2\ell+\cancel{m}-\cancel{m}}{2\ell}$ or $x=\dfrac{2\ell+m+m}{2\ell}$

$x=\dfrac{\cancel{2\ell}}{\cancel{2\ell}}$ or $x=\dfrac{2\ell+2m}{2\ell}$

$x=1$ or $x=\dfrac{\cancel{2}(\ell+m)}{\cancel{2}\ell}$

$$x=\frac{\ell+m}{\ell}$$

Solution set is $\left\{1,\ \dfrac{\ell+m}{\ell}\right\}$