

EXERCISE 1.4

Solve the following equations:

Q.1 $2x + 5 = \sqrt{7x + 16}$

Solution: $2x + 5 = \sqrt{7x + 16}$ (i)

Squaring both sides

$$(2x + 5)^2 = (\sqrt{7x + 16})^2$$

$$(2x)^2 + (5)^2 + 2(2x)(5) = 7x + 16$$

$$4x^2 + 25 + 20x = 7x + 16$$

$$4x^2 + 20x - 7x + 25 - 16 = 0$$

$$4x^2 + 13x + 9 = 0$$

$$4x^2 + 4x + 9x + 9 = 0$$

$$4x(x + 1) + 9(x + 1) = 0$$

$$(x + 1)(4x + 9) = 0$$

$$x + 1 = 0 \quad \text{or} \quad 4x + 9 = 0$$

$$x = -1 \quad \text{or} \quad 4x = -9$$

$$x = -1 \quad \text{or} \quad x = \frac{-9}{4}$$

Checking

Putting $x = -1$ in the equation (i) we have

$$2x + 5 = \sqrt{7x + 16}$$

$$2(-1) + 5 = \sqrt{7(-1) + 16}$$

$$-2 + 5 = \sqrt{-7 + 16}$$

$$3 = \sqrt{9}$$

$$3 = 3 \quad \text{which is true}$$

Putting $x = \frac{-9}{4}$ in the Equation (i) we have

$$2x + 5 = \sqrt{7x + 16}$$

$$2\left(\frac{-9}{4}\right) + 5 = \sqrt{7\left(\frac{-9}{4}\right) + 16}$$

$$\frac{-18}{4} + 5 = \sqrt{\frac{-63}{4} + 16}$$

$$\frac{-18 + 20}{4} = \sqrt{\frac{-63 + 64}{4}}$$

$$\frac{2}{4} = \sqrt{\frac{1}{4}} \quad \Rightarrow \quad \frac{1}{2} = \frac{1}{2} \quad \text{which is true}$$

So, the solution set is $\left\{-1, \frac{-9}{4}\right\}$

Q.2 $\sqrt{x + 3} = 3x - 1$

Solution: $\sqrt{x + 3} = 3x - 1$ (i)

Squaring both sides

$$(\sqrt{x + 3})^2 = (3x - 1)^2$$

$$x + 3 = (3x)^2 + (1)^2 - 2(3x)(1)$$

$$x + 3 = 9x^2 + 1 - 6x$$

$$9x^2 - 6x - x + 1 - 3 = 0$$

$$9x^2 - 7x - 2 = 0$$

$$9x^2 - 9x + 2x - 2 = 0$$

$$9x(x - 1) + 2(x - 1) = 0$$

$$(x - 1)(9x + 2) = 0$$

$$x - 1 = 0 \quad \text{or} \quad 9x + 2 = 0$$

$$x = 1 \quad \text{or} \quad 9x = -2$$

$$\text{or} \quad x = \frac{-2}{9}$$

Checking:

Putting $x = 1$ in the equation (i) we have

$$\sqrt{x + 3} = 3x - 1$$

$$\sqrt{1 + 3} = 3(1) - 1$$

$$\sqrt{4} = 3 - 1$$

$$2 = 2 \quad \text{which is true}$$

Putting $x = \frac{-2}{9}$ in the equation (i) we have

$$\sqrt{x + 3} = 3x - 1$$

$$\sqrt{\frac{-2}{9} + 3} = 3\left(\frac{-2}{9}\right) - 1$$

$$\sqrt{\frac{-2 + 27}{9}} = \frac{-6}{9} - 1$$

$$\sqrt{\frac{25}{9}} = \frac{-6 - 9}{9}$$

$$\frac{5}{3} = \frac{-15}{3}$$

$$\frac{5}{3} = \frac{-5}{3} \quad \text{which is not true}$$

As $\frac{-2}{9}$ is an extraneous root

So, the solution set is $\{1\}$

Q.3 $4x = \sqrt{13x+14} - 3$

Solution: $4x = \sqrt{13x+14} - 3 \dots\dots\dots(i)$

$4x + 3 = \sqrt{13x+14}$

Squaring both sides

$(4x + 3)^2 = (\sqrt{13x+14})^2$

$(4x)^2 + (3)^2 + 2(4x)(3) = 13x + 14$

$16x^2 + 9 + 24x = 13x + 14$

$16x^2 + 9 + 24x - 13x - 14 = 0$

$16x^2 + 11x - 5 = 0$

$16x^2 + 16x - 5x - 5 = 0$

$16x(x+1) - 5(x+1) = 0$

$(x+1)(16x-5) = 0$

$x+1 = 0$ or $16x-5 = 0$

$x = -1$ or $16x = 5$

$x = -1$ or $x = \frac{5}{16}$

Checking

Putting $x = -1$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4(-1) = \sqrt{13(-1)+14} - 3$

$-4 = \sqrt{-13+14} - 3$

$-4 = \sqrt{1} - 3$

$-4 = 1 - 3$

$-4 \neq -2$

So -1 is an extraneous root

Putting $x = \frac{5}{16}$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right)+14} - 3$

$\frac{20}{16} = \sqrt{\frac{65}{16}+14} - 3$

$\frac{5}{4} = \sqrt{\frac{65+224}{16}} - 3$

$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$

$\frac{5}{4} = \frac{17}{4} - 3$

$\frac{5}{4} = \frac{17-12}{4}$

$\frac{5}{4} = \frac{5}{4}$

which is true

So, the solution set is $\left\{\frac{5}{16}\right\}$

Q.4 $\sqrt{3x+100} - x = 4$

Solution: $\sqrt{3x+100} - x = 4 \dots\dots\dots(i)$

$\sqrt{3x+100} = 4 + x$

Squaring both sides

$(\sqrt{3x+100})^2 = (4+x)^2$

$3x+100 = (4)^2 + (x)^2 + 2(4)(x)$

$3x+100 = 16 + x^2 + 8x$

$x^2 + 8x + 16 - 3x - 100 = 0$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

$x(x+12) - 7(x+12) = 0$

$(x+12)(x-7) = 0$

$x+12 = 0$ or $x-7 = 0$

$x = -12$ or $x = 7$

Checking

Putting $x = -12$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(-12)+100} - (-12) = 4$

$\sqrt{-36+100} + 12 = 4$

$\sqrt{64} + 12 = 4$

$8 + 12 = 4$

$20 \neq 4$

So -12 is an extraneous root

Putting $x = 7$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(7)+100} - 7 = 4$

$\sqrt{21+100} - 7 = 4$

$\sqrt{121} - 7 = 4$

$11 - 7 = 4$

$4 = 4$

which is true

So, the solution set is $\{7\}$

Q.5 $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$

Solution: $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$... (i)

Squaring on both sides

$$(\sqrt{x+5} + \sqrt{x+21})^2 = (\sqrt{x+60})^2$$

$$(\sqrt{x+5})^2 + (\sqrt{x+21})^2 + 2(\sqrt{x+5})(\sqrt{x+21}) = x+60$$

$$x+5+x+21+2(\sqrt{x+5})(\sqrt{x+21}) = x+60$$

$$2x+26+2\sqrt{x^2+21x+5x+105} = x+60$$

$$2\sqrt{x^2+26x+105} = x+60-2x-26$$

$$2\sqrt{x^2+26x+105} = -x+34$$

$$2\sqrt{x^2+26x+105} = 34-x$$

Again squaring both sides

$$(2\sqrt{x^2+26x+105})^2 = (34-x)^2$$

$$4(x^2+26x+105) = (34)^2 + x^2 - 2(34)(x)$$

$$4x^2 + 104x + 420 = 1156 + x^2 - 68x$$

$$4x^2 + 104x + 420 - 1156 - x^2 + 68x = 0$$

$$3x^2 + 172x - 736 = 0$$

$$a = 3, b = 172, c = -736$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-172 \pm \sqrt{(172)^2 - 4(3)(-736)}}{2(3)}$$

$$x = \frac{-172 \pm \sqrt{29584 + 8832}}{6}$$

$$x = \frac{-172 \pm \sqrt{38416}}{6}$$

$$x = \frac{-172 \pm 196}{6}$$

$$x = \frac{-172-196}{6} \text{ or } x = \frac{-172+196}{6}$$

$$x = \frac{-368}{6} \text{ or } x = \frac{24}{6}$$

$$x = \frac{-184}{3} \text{ or } x = 4$$

Checking:

Putting $x = 4$ in equation (i)

$$\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$$

$$\sqrt{4+5} + \sqrt{4+21} = \sqrt{4+60}$$

$$\sqrt{9} + \sqrt{25} = \sqrt{64}$$

$$3+5=8$$

$$8=8$$

which is true

Putting $x = \frac{-184}{3}$

$$\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$$

$$\sqrt{\frac{-184}{3}+5} + \sqrt{\frac{-184}{3}+21} = \sqrt{\frac{-184}{3}+60}$$

$$\sqrt{\frac{-184+15}{3}} + \sqrt{\frac{-184+63}{3}} = \sqrt{\frac{-184+180}{3}}$$

$$\sqrt{\frac{-169}{3}} + \sqrt{\frac{-121}{3}} = \sqrt{\frac{-4}{3}}$$

$$\sqrt{\frac{-1 \times 169}{3}} + \sqrt{\frac{-1 \times 121}{3}} = \sqrt{\frac{-1 \times 4}{3}}$$

$$\frac{13i}{\sqrt{3}} + \frac{11i}{\sqrt{3}} = \frac{2i}{\sqrt{3}} \quad \because i = \sqrt{-1}$$

$$\frac{13i+11i}{\sqrt{3}} = \frac{2i}{\sqrt{3}}$$

$$\frac{24i}{\sqrt{3}} = \frac{2i}{\sqrt{3}}$$

Which is not true

As $x = \frac{-184}{3}$ is extraneous root

so solution set is $\{4\}$

Q.6 $\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$

Solution: $\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$ (i)

Squaring both sides

$$(\sqrt{x+1} + \sqrt{x-2})^2 = (\sqrt{x+6})^2$$

$$(\sqrt{x+1})^2 + (\sqrt{x-2})^2 + 2(\sqrt{x+1})(\sqrt{x-2}) = x+6$$

$$x+1+x-2+2\sqrt{(x+1)(x-2)} = x+6$$

$$2x-1+2\sqrt{x^2-2x+1x-2} = x+6$$

$$2\sqrt{x^2-x-2} = x+6-2x+1$$

$$2\sqrt{x^2-x-2} = 7-x$$

Again squaring both sides

$$(2\sqrt{x^2 - x - 2})^2 = (7 - x)^2$$

$$4(x^2 - x - 2) = (7)^2 + (x)^2 - 2(7)(x)$$

$$4x^2 - 4x - 8 = 49 + x^2 - 14x$$

$$4x^2 - x^2 - 4x + 14x - 8 - 49 = 0$$

$$3x^2 + 10x - 57 = 0$$

$$3x^2 + 19x - 9x - 57 = 0$$

$$x(3x + 19) - 3(3x + 19) = 0$$

$$(3x + 19)(x - 3) = 0$$

$$3x + 19 = 0 \quad \text{or} \quad x - 3 = 0$$

$$3x = -19 \quad \text{or} \quad x = 3$$

$$x = \frac{-19}{3}$$

Checking

Putting $x = 3$ in the equation (i)

$$\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$$

$$\sqrt{3+1} + \sqrt{3-2} = \sqrt{3+6}$$

$$\sqrt{4} + \sqrt{1} = \sqrt{9}$$

$$2 + 1 = 3$$

$$3 = 3 \quad \text{which is true}$$

Putting $x = \frac{-19}{3}$ in the equation (i)

$$\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$$

$$\sqrt{\frac{-19}{3} + 1} + \sqrt{\frac{-19}{3} - 2} = \sqrt{\frac{-19}{3} + 6}$$

$$\sqrt{\frac{-19+3}{3}} + \sqrt{\frac{-19-6}{3}} = \sqrt{\frac{-19+18}{3}}$$

$$\sqrt{\frac{-1 \times 16}{3}} + \sqrt{\frac{-1 \times 25}{3}} = \sqrt{\frac{-1 \times 1}{3}}$$

$$\frac{4i}{\sqrt{3}} + \frac{5i}{\sqrt{3}} = \frac{li}{\sqrt{3}} \quad \because i = \sqrt{-1}$$

$$\frac{4i + 5i}{\sqrt{3}} = \frac{li}{\sqrt{3}}$$

$$\frac{9i}{\sqrt{3}} = \frac{li}{\sqrt{3}} \quad \text{which is not true}$$

As $x = \frac{-19}{3}$ is an extraneous root

So the solution set is $\{3\}$

$$\text{Q.7} \quad \sqrt{11-x} + \sqrt{6-x} = \sqrt{27-x}$$

$$\text{Solution: } \sqrt{11-x} + \sqrt{6-x} = \sqrt{27-x} \dots\dots(i)$$

Squaring both sides

$$(\sqrt{11-x} + \sqrt{6-x})^2 = (\sqrt{27-x})^2$$

$$(\sqrt{11-x})^2 + (\sqrt{6-x})^2 + 2(\sqrt{11-x})(\sqrt{6-x}) = 27-x$$

$$11-x+6-x+2\sqrt{(11-x)(6-x)} = 27-x$$

$$17-2x+2\sqrt{66-11x-6x+x^2} = 27-x$$

$$2\sqrt{66-17x+x^2} = 27-x-17+2x$$

$$2\sqrt{66-17x+x^2} = 10+x$$

Again squaring both sides

$$(2\sqrt{66-17x+x^2})^2 = (10+x)^2$$

$$4(66-17x+x^2) = (10)^2 + (x)^2 + 2(10)(x)$$

$$264 - 68x + 4x^2 = 100 + x^2 + 20x$$

$$4x^2 - x^2 - 68x - 20x + 264 - 100 = 0$$

$$3x^2 - 88x + 164 = 0$$

By applying quadratic formula

$$a = 3, \quad b = -88, \quad c = 164$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-88) \pm \sqrt{(-88)^2 - 4(3)(164)}}{2(3)}$$

$$x = \frac{88 \pm \sqrt{7744 - 1968}}{6}$$

$$x = \frac{88 \pm \sqrt{5776}}{6}$$

$$x = \frac{88-76}{6} \quad \text{or} \quad x = \frac{88+76}{6}$$

$$x = \frac{2}{6} \quad \text{or} \quad x = \frac{164}{6}$$

$$x = 2 \quad \text{or} \quad x = \frac{82}{3}$$

Checking

Putting $x = 2$ in the equation (i)

$$\sqrt{11-x} + \sqrt{6-x} = \sqrt{27-x}$$

$$\sqrt{11-2} + \sqrt{6-2} = \sqrt{27-2}$$

$$\sqrt{9} + \sqrt{4} = \sqrt{25}$$

$$3 + 2 = 5$$

$$5 = 5 \quad \text{which is true}$$

Putting $x = \frac{82}{3}$ in the equation (i)

$$\sqrt{11-x} + \sqrt{6-x} = \sqrt{27-x}$$

$$\sqrt{11-\frac{82}{3}} + \sqrt{6-\frac{82}{3}} = \sqrt{27-\frac{82}{3}}$$

$$\sqrt{\frac{33-82}{3}} + \sqrt{\frac{18-82}{3}} = \sqrt{\frac{81-82}{3}}$$

$$\sqrt{\frac{-1 \times 49}{3}} + \sqrt{\frac{-1 \times 64}{3}} = \sqrt{\frac{-1 \times 1}{3}}$$

$$\frac{7i}{\sqrt{3}} + \frac{8i}{\sqrt{3}} = \frac{i}{\sqrt{3}} \quad \therefore i = \sqrt{-1}$$

$$\frac{7i+8i}{\sqrt{3}} = \frac{i}{\sqrt{3}}$$

$$\frac{15i}{\sqrt{3}} = \frac{i}{\sqrt{3}} \quad \text{which is not true}$$

As $\frac{82}{3}$ is an extraneous root

So, the solution set is $\{2\}$

Q.8 $\sqrt{4a+x} - \sqrt{a-x} = \sqrt{a}$

Solution: $\sqrt{4a+x} - \sqrt{a-x} = \sqrt{a}$ (i)

Squaring both sides

$$(\sqrt{4a+x} - \sqrt{a-x})^2 = (\sqrt{a})^2$$

$$(\sqrt{4a+x})^2 + (\sqrt{a-x})^2 - 2(\sqrt{4a+x})(\sqrt{a-x}) = a$$

$$4a + \cancel{x} + a - \cancel{x} - 2\sqrt{(4a+x)(a-x)} = a$$

$$5a - 2\sqrt{4a^2 - 4ax + ax - x^2} = a$$

$$-2\sqrt{4a^2 - 3ax - x^2} = a - 5a$$

$$-2\sqrt{4a^2 - 3ax - x^2} = -4a$$

$$\sqrt{4a^2 - 3ax - x^2} = \frac{-\cancel{4}^2 a}{-\cancel{2}}$$

$$\sqrt{4a^2 - 3ax - x^2} = 2a$$

Again squaring both sides

$$(\sqrt{4a^2 - 3ax - x^2})^2 = (2a)^2$$

$$4a^2 - 3ax - x^2 = 4a^2$$

$$4a^2 - 3ax - x^2 = 4a^2$$

$$4\cancel{a^2} - 4\cancel{a^2} = 3ax + x^2$$

$$\Rightarrow 3ax + x^2 = 0$$

$$x(3a + x) = 0$$

$$x = 0 \quad \text{or} \quad 3a + x = 0$$

$$x = 0 \quad \text{or} \quad x = -3a$$

Checking:

Putting $x = 0$ in the equation (i)

$$\sqrt{4a+x} - \sqrt{a-x} = \sqrt{a}$$

$$\sqrt{4a+0} - \sqrt{a-0} = \sqrt{a}$$

$$\sqrt{4a} - \sqrt{a} = \sqrt{a}$$

$$2\sqrt{a} - \sqrt{a} = \sqrt{a}$$

$$\sqrt{a} = \sqrt{a} \quad \text{which is true}$$

Putting $x = -3a$ in the equation (i)

$$\sqrt{4a+x} - \sqrt{a-x} = \sqrt{a}$$

$$\sqrt{4a-3a} - \sqrt{a+3a} = \sqrt{a}$$

$$\sqrt{a} - \sqrt{4a} = \sqrt{a}$$

$$\sqrt{a} - 2\sqrt{a} = \sqrt{a}$$

$$-\sqrt{a} = \sqrt{a} \quad \text{which is not true}$$

As $-3a$ is extraneous root

So, the solution set is $\{0\}$

Q.9 $\sqrt{x^2 + x + 1} - \sqrt{x^2 + x - 1} = 1$

Solution: $\sqrt{x^2 + x + 1} - \sqrt{x^2 + x - 1} = 1 \dots\dots(i)$

Let $x^2 + x = y \dots\dots(ii)$

Put it in equation (i)

$$\sqrt{y+1} - \sqrt{y-1} = 1$$

Squaring both sides

$$(\sqrt{y+1} - \sqrt{y-1})^2 = (1)^2$$

$$(\sqrt{y+1})^2 + (\sqrt{y-1})^2 - 2(\sqrt{y+1})(\sqrt{y-1}) = 1$$

$$y + \cancel{1} + y - \cancel{1} - 2\sqrt{(y+1)(y-1)} = 1$$

$$2y - 2\sqrt{y^2 - 1} = 1$$

$$\Rightarrow 2y - 1 = 2\sqrt{y^2 - 1}$$

Again squaring both sides

$$(2y - 1)^2 = (2\sqrt{y^2 - 1})^2$$

$$(2y)^2 + (1)^2 - 2(2y)(1) = 4(y^2 - 1)$$

$$4y^2 + 1 - 4y = 4y^2 - 4$$

$$4\cancel{y^2} + 1 - 4y = 4\cancel{y^2} - 4$$

$$1 - 4y = -4$$

$$1 + 4 = 4y$$

$$5 = 4y$$

$$\Rightarrow 4y = 5$$

From equation (ii) Put $y = x^2 + x$

$$4y = 5$$

$$4(x^2 + x) = 5$$

$$4x^2 + 4x = 5$$

$$4x^2 + 4x - 5 = 0$$

Solving by quadratic formula

$$a = 4, \quad b = 4, \quad c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16 + 80}}{8}$$

$$x = \frac{-4 \pm \sqrt{96}}{8}$$

$$x = \frac{-4 \pm \sqrt{16 \times 6}}{8}$$

$$x = \frac{-4 \pm 4\sqrt{6}}{8}$$

$$x = \frac{\cancel{4}(-1 \pm \sqrt{6})}{\cancel{8}_2}$$

$$x = \frac{-1 \pm \sqrt{6}}{2}$$

The solution set is $\left\{ \frac{-1 \pm \sqrt{6}}{2} \right\}$

Q.10 $\sqrt{x^2 + 3x + 8} + \sqrt{x^2 + 3x + 2} = 3$

Solution: $\sqrt{x^2 + 3x + 8} + \sqrt{x^2 + 3x + 2} = 3 \dots(i)$

Let $x^2 + 3x = y \dots\dots(ii)$

Put it in equation (i)

$$\sqrt{y+8} + \sqrt{y+2} = 3$$

Squaring on both sides

$$(\sqrt{y+8} + \sqrt{y+2})^2 = (3)^2$$

$$(\sqrt{y+8})^2 + (\sqrt{y+2})^2 + 2(\sqrt{y+8})(\sqrt{y+2}) = 9$$

$$y + 8 + y + 2 + 2\sqrt{y^2 + 2y + 8y + 16} = 9$$

$$2y + 10 + 2\sqrt{y^2 + 10y + 16} = 9$$

$$2\sqrt{y^2 + 10y + 16} = 9 - 2y - 10$$

$$2\sqrt{y^2 + 10y + 16} = -2y - 1$$

$$2\sqrt{y^2 + 10y + 16} = -(2y + 1)$$

Again squaring on both sides

$$(2\sqrt{y^2 + 10y + 16})^2 = [-(2y + 1)]^2$$

$$4(y^2 + 10y + 16) = (2y + 1)^2$$

$$4y^2 + 40y + 64 = (2y)^2 + (1)^2 + 2(2y)(1)$$

$$4y^2 + 40y + 64 = 4y^2 + 1 + 4y$$

$$40y + 64 = 1 + 4y$$

$$40y - 4y = 1 - 64$$

$$36y = -63$$

From equation (ii) Put $y = x^2 + 3x$

$$36(x^2 + 3x) = -63$$

$$36x^2 + 108x + 63 = 0$$

Solving by quadratic formula

$$a = 36, \quad b = 108, \quad c = 63$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-108 \pm \sqrt{(108)^2 - 4(36)(63)}}{2(36)}$$

$$= \frac{-108 \pm \sqrt{11664 - 9072}}{72}$$

$$= \frac{-108 \pm \sqrt{2592}}{72}$$

$$x = \frac{-108 \pm \sqrt{4 \times 4 \times 9 \times 9 \times 2}}{72}$$

$$x = \frac{-108 \pm \sqrt{4^2 \times 9^2 \times 2}}{72}$$

$$x = \frac{-108 \pm 4 \times 9 \sqrt{2}}{72}$$

$$x = \frac{-108 \pm 36\sqrt{2}}{72}$$

$$x = \frac{36(-3 \pm \sqrt{2})}{72}$$

$$x = \frac{-3 \pm \sqrt{2}}{2}$$

So, the solution set is $\left\{ \frac{-3 \pm \sqrt{2}}{2} \right\}$

$$\text{Q.11 } \sqrt{x^2 + 3x + 9} + \sqrt{x^2 + 3x + 4} = 5$$

$$\text{Solution: } \sqrt{x^2 + 3x + 9} + \sqrt{x^2 + 3x + 4} = 5 \dots (i)$$

$$\text{Let } x^2 + 3x = y \dots (ii)$$

Put it in equation (i)

$$\sqrt{y+9} + \sqrt{y+4} = 5$$

Squaring both sides

$$(\sqrt{y+9} + \sqrt{y+4})^2 = 5^2$$

$$(\sqrt{y+9})^2 + (\sqrt{y+4})^2 + 2(\sqrt{y+9})(\sqrt{y+4}) = 25$$

$$y+9 + y+4 + 2\sqrt{(y+9)(y+4)} = 25$$

$$2y+13 + 2\sqrt{y^2+4y+9y+36} = 25$$

$$2\sqrt{y^2+13y+36} = 25-2y-13$$

$$2\sqrt{y^2+13y+36} = 12-2y$$

$$\cancel{2}\sqrt{y^2+13y+36} = \cancel{2}(6-y)$$

$$\sqrt{y^2+13y+36} = 6-y$$

Again taking squaring both sides

$$(\sqrt{y^2+13y+36})^2 = (6-y)^2$$

$$y^2+13y+36 = (6)^2 + (y)^2 - 2(6)(y)$$

$$\cancel{y^2} + 13y + \cancel{36} = \cancel{36} + \cancel{y^2} - 12y$$

$$13y = -12y$$

$$13y + 12y = 0$$

$$25y = 0$$

$$y = \frac{0}{25}$$

$$y = 0$$

From equation (ii) Put $x^2 + 3x = y$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x = 0$$

$$\text{or } x+3 = 0$$

$$x = -3$$

So, the solution set is $\{-3, 0\}$