Exercise 10.2

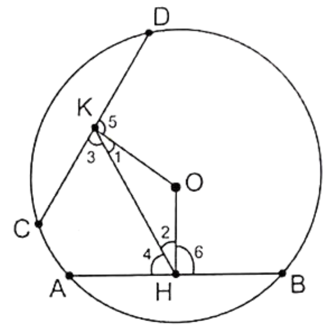
Q. 1 **$\overline{AB}$ and $\overline{CD}$ are two equal chords in a circle with centre O. H and K are respectively the mid points of the chords. Prove that $\overline{HK}$ makes equal angles with $\overline{AB}$ and $\overline{CD}$.**

Given:

A circle with centre 'O'. Two chords such that

$m\overline{AB} = m\overline{CD}$. H and K are mid points of chords AB and CD respectively.

H is joined with K



To Prove:

(i) $m\angle AHK = m\angle CKH$

(ii) $m\angle BHK = m\angle DKH$

Proof:

Statements	Reasons
In ΔHOK	
$m\overline{OH} = m\overline{OK}$	Two equal chords are equidistant from the center.
$\therefore$ $m\angle 1 = m\angle 2$ .... (i)	Angles opposite to the equal line segments
And $m\angle 5 = m\angle 6$ .... (ii)	Each 90^o
$m\angle 1 + m\angle 5 = m\angle 2 + m\angle 6$	Adding (i) and (ii)
Thus, $m\angle DKH = m\angle BHK$	
or $m\angle BHK = m\angle DKH$ Proved	
$m\angle AHO = m\angle CKO$	Each 90^o
$m\angle 2 + m\angle 4 = m\angle 1 + m\angle 3$	
But $m\angle 2 = m\angle 1$	Proved in (i)
$\cancel{m\angle 1} + m\angle 4 = \cancel{m\angle 1} + m\angle 3$	
$m\angle 4 = m\angle 3$	By cancellation property
$m\angle AHK = m\angle CKH$	

Q.2 **The radius of a circle is 2.5 cm. $\overline{AB}$ and $\overline{CD}$ are two chords 3.9cm apart. If $m\overline{AB} = 1.4$ cm, then measure the other chord.**

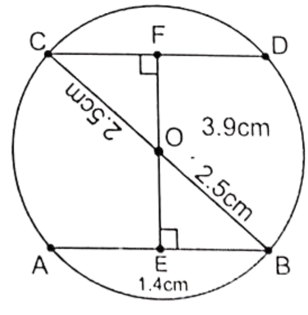
Given:

O is the centre of a circle.

(i) $m\overline{OB} = m\overline{OC} = 2.5$cm

(ii) $m\overline{AB} = 1.4$cm

(iii) $m\overline{EF} = 3.9$cm



To Find:

$m\overline{CD} = ?$

Construction:

Join O with B and C.

Calculations:

Steps	Reasons
In ΔOEB	Given
$m\overline{EB} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(1.4cm) = 0.7\,cm$ ....(i)	
$m\overline{OB} = 2.5cm$ ....(ii)	
$(m\overline{OB})^2 = (m\overline{OE})^2 + (m\overline{EB})^2$	By Pythagoras theorem in right angled ΔOEB
$(2.5cm)^2 = (m\overline{OE})^2 + (0.7cm)^2$	From (i) and (ii)
$\Rightarrow (m\overline{OE})^2 = (2.5cm)^2 - (0.7cm)^2$	
$(m\overline{OE})^2 = 6.25cm^2 - 0.49cm^2$	
$(m\overline{OE})^2 = 5.76cm^2$	
$\sqrt{(m\overline{OE})^2} = \sqrt{5.76cm^2}$	
$m\overline{OE} = 2.4cm$ .... (iii)	
Now	
$m\overline{OF} = m\overline{EF} - m\overline{OE}$	
$m\overline{OF} = 3.9cm - 2.4cm$	
$m\overline{OF} = 1.5cm$ .... (iv)	
In right angled triangle ΔOCF	
$(m\overline{OC})^2 = (m\overline{OF})^2 + (m\overline{CF})^2$	
$\Rightarrow (m\overline{CF})^2 = (2.5cm)^2 - (1.5cm)^2$	From (iv)
$(m\overline{CF})^2 = 6.25cm^2 - 2.25cm^2$	
$(m\overline{CF})^2 = 4cm^2$	
$\therefore \sqrt{(m\overline{CF})^2} = \sqrt{4cm^2}$	
$(m\overline{CF}) = 2\,cm$ .... (v)	
$\therefore (m\overline{CD}) = 2(m\overline{CF})$	$\because m\overline{CF} = \frac{1}{2} m\overline{CD}$
$m\overline{CD} = 2(2cm)$	From (v)
$m\overline{CD} = 4cm$	

Q.3. **The radii of two intersecting circles are 10cm and 8cm. If the length of their common chord is 6cm then find the distance between the centres.**

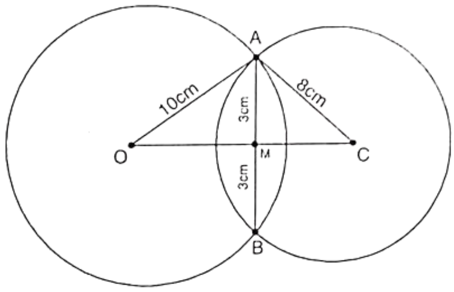
Solution:

Given: Two intersecting circles with centers O and C having radius $m\overline{OA} = 10cm$, radius $m\overline{AC} = 8cm$ respectively.

Length of common chord $m\overline{AB} = 6cm$

To Find:

Distance between the centers $m\overline{OC} = ?$



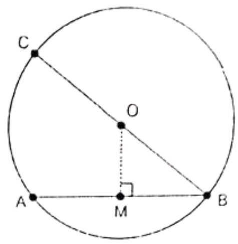
Construction:

Join the point A to the centers O and C. Joint O to C which meets the chord $\overline{AB}$ at its midpoint.

Calculations:

Steps	Reasons
$\overline{OM} \perp \overline{AB}$	
$m\overline{AM} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(6cm) = 3cm$ ....(i)	$\perp$ from the center to the chord bisect it.
By Pythagoras theorem	
$(m\overline{OM})^2 = (m\overline{OA})^2 - (m\overline{AM})^2$	
$= (10cm)^2 - (3cm)^2$	From given and from (i)
$= 100cm^2 - 9cm^2$	
$(m\overline{OM})^2 = 91\,cm^2$	
$\sqrt{(m\overline{OM})^2} = \sqrt{91cm^2}$	
$m\overline{OM} = 9.54cm$ ....(ii)	
In ΔAMC	
By Pythagoras theorem	
$(m\overline{MC})^2 = (m\overline{CA})^2 - (m\overline{AM})^2$	
$= (8cm)^2 - (3cm)^2$	$m\overline{CA} = 8cm$, $m\overline{AM} = 3cm$
$= 64cm^2 - 9cm^2$	
$(m\overline{MC})^2 = 55\ cm^2$	
$\sqrt{(m\overline{MC})^2} = \sqrt{55cm^2}$	
$m\overline{MC} = 7.42\,cm$ ....(iii)	
We know that the distance between the centers	
$(m\overline{OC}) = (m\overline{OM}) + (m\overline{MC})$	From (ii) and (iii)
$= 9.54mc + 7.42cm$	
$= 16.96cm$	
Distance between the centres is 16.96cm	

Q.4. Show that greatest chord in a circle is its diameter.



Given: O be the centre of the circle, $m\overline{BC}$ is central chord and $\overline{AB}$ be any chord of the circle

To prove: Central chord $m\overline{CB}$ > Any chord $m\overline{AB}$

Construction: Draw $\overline{OM} \perp \overline{AB}$ to make right angled triangle OMB.

Proof:

Statements	Reasons
In right angled triangle OMB $(m\overline{OB})^2 = (m\overline{OM})^2 + (m\overline{MB})^2$	By Pythagoras theorem
It means $m\overline{OB} > m\overline{MB}$ $\therefore \quad 2(m\overline{OB}) > 2(m\overline{MB})$	The length of hypotenuse is greater than the length of other two sides.
As $2(m\overline{OB})$ is length of the central chord and $2(m\overline{MB})$ is length of the chord $\overline{AB}$ thus, Central chord $m\overline{CB}$ > Any chord $m\overline{AB}$.	
It means central chord of the circle i.e. diameter is greater than any other chord of the circle, which proved that the greatest chord in a circle is its diameter.	