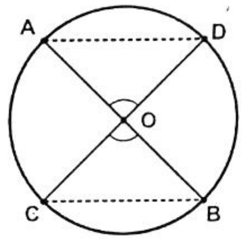
EXERCISE 11.1

Q.1 In a circle two equal diameters $\overline{AB}$ and $\overline{CD}$ intersect each other. Prove that $m\overline{AD} = m\overline{BC}$.

Given: A circle with centre "O". Two diameters $\overline{AB}$ and $\overline{BC}$, intersecting at point O.



To Prove: $m\overline{AD} = m\overline{BC}$

Construction:

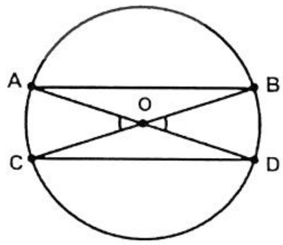
Join A to D and C to B

Proof:

Statements	Reasons
In $\Delta AOD \leftrightarrow \Delta BOC$	
$\overline{OA} \cong \overline{OB}$	Radii of the same circle
$\angle AOD \cong \angle BOC$	Vertical angles are congruent
$\overline{OD} \cong \overline{OC}$	Radii of the same circle
$\therefore \Delta AOD \cong \Delta BOC$	S. A. S $\cong$ S. A. S
$\overline{AD} \cong \overline{BC}$	Corresponding sides of congruent triangle
Or $m\overline{AD} = \overline{mBC}$	

Q.2. In a circle prove that the arcs between two parallel and equal chords are equal.

Given: A circle with centre O. Two chords $\overline{AB}$ and $\overline{CD}$ Such that $\overline{AB} \parallel \overline{CD}$ and $m\overline{AB} = m\overline{CD}$



To Prove: $m\widehat{AC} = m\widehat{BD}$

Construction: Join A to D and B to C. Such that $\overline{AD}$ and $\overline{CD}$ intersect each other at central point O.

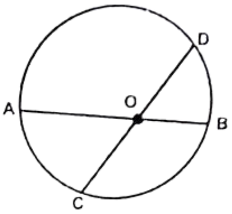
Proof:

Statements	Reasons
$\overline{AD}$ and $\overline{BC}$ are line segment intersecting at centre O.	
$\angle AOC$ and $\angle BOD$ are central angles.	Angle subtended at centre.
$m\angle AOC = m\angle BOD$	Vertical angles
$m\widehat{AC} = m\widehat{BB}$	Within the same circle arcs opposite to the equal central angles are equal.

Q.3. Give a geometric proof that a pair of bisecting chords are the diameters of a circle.

Given: A circle and two chords $\overline{AB}$ and $\overline{CD}$ bisecting each other at point O. i.e.

$m\overline{AO} = m\overline{OB}$ and $m\overline{CO} = m\overline{OD}$



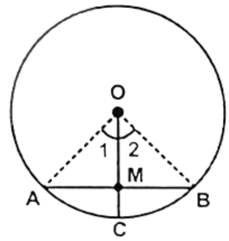
To Prove: Chords $\overline{AB}$ and $\overline{CD}$ are diameters.

Proof:

Statements	Reasons
$m\overline{AB} = m\overline{CD}$ ....(i)	Two chords can bisect each other only when they are equal (given)
$\therefore$ O is the mid point of $\overline{AB}$ and $\overline{CD}$	Given
$m\overline{AO} = m\overline{BO} = \frac{1}{2} m\overline{AB}$ ....(ii)	
$m\overline{DO} = m\overline{CO} = \frac{1}{2} m\overline{CD}$ ....(iii)	
$m\overline{AO} = m\overline{BO} = m\overline{CO} = m\overline{DO}$ ....(iv)	From (i), (ii) and (iii)
The points of circle A, B, C and D are equidistant form the fixed point "O".	From (iv)
This fixed pint O is the centre of the circle having the points A, B, C and D.	By definition
As chords $\overline{AB}$ and $\overline{CD}$ pass through the centre "O" therefore chords $\overline{AB}$ and $\overline{CD}$ are diameters.	

Q.4. If C is the midpoint of an arc ACB in a circle with centre O. Show that line segment OC bisects the chord AB.

Given: A circle with centre "O" $\widehat{ACB}$ is an arc with C as its midpoint and $m\widehat{AC} = m\widehat{CB}$. Center "O" is joined with C such that $\overline{OC}$ meets $\overline{AB}$ at M.



To Prove: $m\overline{AM} = m\overline{BM}$

Construction: Join center "O" with A and B to make central angle AOB.

Proof:

Statements	Reasons
$\angle AOB$ is central angle	Construction
$\therefore$ $m\angle 1 = m\angle 2$ ....(i)	C is the midpoint of $\widehat{ACB}$ (Given)
In $\Delta AOM \longleftrightarrow \Delta BOM$	
$\overline{OM} \cong \overline{OM}$	Common
$\angle 1 \cong \angle 2$	Proved
$\overline{OA} \cong \overline{OB}$	Radii of the same Circle
$\Delta AOM \cong \Delta BOM$	S.A.S ≅ S.A.S
$\overline{AM} \cong \overline{BM}$	Corresponding sides of congruent triangles.
Hence $m\overline{AM} = m\overline{BM}$	