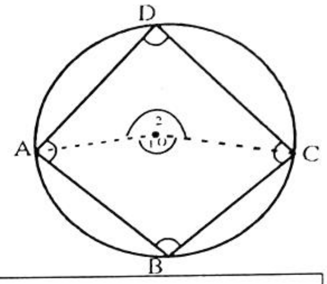
# EXERCISE 12.1

**Q.1 Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.**




**Given:** A circle with centre "O"

ABCD is a cyclic quadrilateral

**To Prove:** $m\angle B + m\angle D = 180^o$

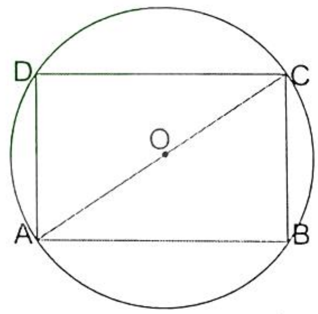
$m\angle BCD + m\angle DAB = 180^o$

**Construction:** Join O with A and C

**Proof:**

| Statements | Reasons |
|---|---|
| $m\angle 1 = 2m\angle D$..........(i) | $\angle 1$, $\angle 2$ are central angles and $\angle D$, $\angle B$ are circum angles in Arcs |
| $m\angle 2 = 2m\angle B$..........(ii) | |
| $m\angle 1 + m\angle 2 = 2m\angle D + 2m\angle B$ | Adding (i) and (ii) |
| $m\angle 1 + m\angle 2 = 2(m\angle D + m\angle B)$ | |
| or $2(m\angle D + m\angle B) = m\angle 1 + m\angle 2$ | By symmetric property |
| $2(m\angle D + m\angle B) = 360^o$ | Sum of all central angles is $360^o$ |
| $m\angle D + m\angle B = \frac{360^o}{2}$ | Dividing by 2 |
| $m\angle D + m\angle B = 180^o$ | |
| Similarly $m\angle BCD + m\angle DAB = 180^o$ | |

**Q.2 Show that parallelogram inscribed in a circle will be a rectangle.**




**Given:** ABCD is a parallelogram inscribed in the circle with centre "O"

$m\overline{AB} = m\overline{DC}$ and $\overline{AB} \parallel \overline{DC}$

$m\overline{AD} = m\overline{BC}$ and $\overline{AD} \parallel \overline{BC}$

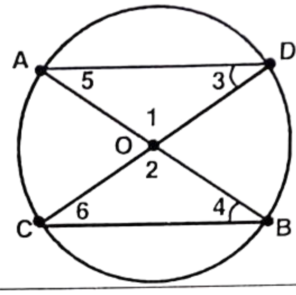
**To Prove:** ABCD is a rectangle

**Construction:** Join A with C

**Proof:**

| Statements | Reasons |
|---|---|
| In $\Delta ABC \leftrightarrow \Delta ADC$ | |
| $m\overline{AC} = m\overline{AC}$ | Common |
| $m\overline{AB} = m\overline{DC}$ | Given |
| $m\overline{BC} = m\overline{AD}$ | Given |
| $\therefore \Delta ABC \cong \Delta ADC$ | S.S. S $\cong$ S. S. S |
| Thus, $m\angle B = m\angle D$........................(i) | Corresponding angles of congruent triangles |
| $m\angle B + m\angle D = 180^o$..........(ii) | Opposite angles of parallelogram |
| $\Rightarrow m\angle B = m\angle D = 90^o$ | From (i) |
| Similarly $m\angle BAD = m\angle BCD = 90^o$ | From (i) and (ii) |
| Hence ABCD is rectangle | |

**Q.3 AOB and COD are two intersecting chords of a circle. Show that Δ's AOD and BOC are equiangular.**




**Given:** In a circle $\overline{AOB}$ and $\overline{COD}$ are two intersecting chords at point O.

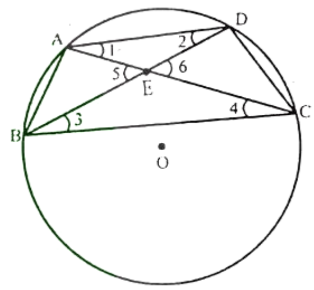
**To Prove:** $\Delta AOD$ and $\Delta BOC$ are equiangular

**Construction:** Join A with C and D. Join B with C and D.

**Proof:**

| Statements | Reasons |
|---|---|
| $m\angle 1 \cong m\angle 2$........(i) | Vertical angles |
| $\overline{AC}$ is chord and angles $\angle 3$, $\angle 4$ are in the same segment. | |
| $\angle 3 \cong \angle 4$............(ii) | |
| Now $\overline{BD}$ is chord and angles $\angle 5$, $\angle 6$ are in the same segments | |
| Therefore $\angle 5 \cong \angle 6$............(iii) | |
| Thus, $\Delta AOD$ and $\Delta BOC$ are equiangular | From (i), (ii) and (iii) |

**Q.4 $\overline{AD}$ and $\overline{BC}$ are two parallel chords of a circle prove that arc AB ≅ are CD and arc AC ≅ arc BD.**




**Given:** A circle with centre "O". Two chords $\overline{AD}$ and $\overline{BC}$ are such that $\overline{AD} \parallel \overline{BC}$.

**To Prove:** arc AB ≅ arc CD and arc AC ≅ arc BD

**Construction:** Join A to B and C. Join D to B and C. $\overline{AC}$ and $\overline{BD}$ intersect each other at point E. some angles are named as $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6$.

**Proof:**

| | Statements | | Reasons |
|---|---|---|---|
| | $m\angle 1 = m\angle 3$ | ...........(i) | Angles inscribed by an arc in the same segment are equal. |
| | $m\angle 2 = m\angle 4$ | ...........(ii) | |
| | $m\angle 1 = m\angle 4$ | ...........(iii) | Alternate angles are congruent $(\overline{AD} \parallel \overline{BC})$ |
| | $m\angle 3 = m\angle 4$ | ...........(iv) | From (i) and (iii) |
| | $m\angle 1 = m\angle 2$ | ...........(v) | From (ii) and (iii) |
| In | $\Delta AEB \leftrightarrow \Delta DEC$ | | |
| | $\overline{AE} \cong \overline{ED}$ | | Side opposite to equal angles (v) |
| | $m\angle 5 = m\angle 6$ | | vertical angles |
| | $\overline{BE} \cong \overline{EC}$ | | Sides opposite to equal angles (iv) |
| ∴ | $\Delta AED \cong \Delta DEC$ | | S.A.S ≅ S.A.S |
| | $\overline{AB} \cong \overline{CD}$ | | Corresponding sides of congruent. |
| Thus | arc AB ≅ arc CD | (Hence Proved) | Arcs corresponding to congruent chords are congruent. |
| | $m\widehat{BC} \cong m\widehat{CB}$ | | |
| | $m\widehat{BA} + m\widehat{AC} = m\widehat{CD} + m\widehat{DB}$ | | Self congruent |
| | $\cancel{m\widehat{AB}} + m\widehat{AC} = \cancel{m\widehat{AB}} + m\widehat{BD}$ | | ∵ arc $\widehat{AB}$ ≅ arc CD proved |
| | $m\widehat{AC} = m\widehat{BD}$ | | |
| or | arc AC ≅ arc BD | (Hence proved) | |