# EXERCISE 2.2

**Q.1 Find the cube roots of $-1, 8, -27, 64$.**

**(i) Cube roots of $-1$**

**Solution:**

Let $x = (-1)^{\frac{1}{3}}$

$x^3 = -1$

$x^3 + 1 = 0$

$x^3 + (1)^3 = 0$

$\because (a^3 + b^3) = (a+b)(a^2 - ab + b^2)$

$(x+1)\left[x^2 - (x)(1) + 1^2\right] = 0$

$(x+1)(x^2 - x + 1) = 0$

$x + 1 = 0$ or

$\boxed{x = -1}$ or $\boxed{x^2 - x + 1 = 0}$

Now we solve $x^2 - x + 1 = 0$ by formula

$ax^2 + bx + c = 0$

$a = 1, \quad b = -1, \; c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2\times 1}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2\times 1}$$

$$= \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

$$= -1\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

$$x = -1\left(\frac{-1 + \sqrt{-3}}{2}\right) \; ; \; x = -1\left(\frac{-1 - \sqrt{-3}}{2}\right)$$

$x = -1\omega$ or $x = -1(\omega^2)$

$x = -\omega$ or $x = -\omega^2$

So, cube roots of $-1$ are $-1, -\omega$ and $-\omega^2$

**(ii) Cube roots of 8**

**Solution:**

Let $x = (8)^{\frac{1}{3}}$

$x^3 = 8$

$x^3 - 8 = 0$

$x^3 - 2^3 = 0$

$\because (a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$(x-2)\left[x^2 + (x)(2) + 2^2\right] = 0$

$(x-2)(x^2 + 2x + 4) = 0$

$x - 2 = 0$ or $x^2 + 2x + 4 = 0$

$x = 2$

Now we solve $x^2 + 2x + 4 = 0$ by formula

$a = 1, \; b = 2, \; c = 4$

$$x = \frac{-2 \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2\times 1}$$

$$= \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm \sqrt{4\times(-3)}}{2}$$

$$= \frac{-2 \pm \sqrt{4}\sqrt{-3}}{2}$$

$$= \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$x = \frac{2\left(-1 \pm \sqrt{-3}\right)}{2}$$

$$x = 2\left(\frac{-1 + \sqrt{-3}}{2}\right), \quad x = 2\left(\frac{-1 - \sqrt{3}}{2}\right)$$

$x = 2\omega$ , $x = 2\omega^2$

So cube roots of 8 are $2, \; 2\omega, \; 2\omega^2$

(iii) **Cube roots of –27**

**Solution:**

Let $x=(-27)^{\frac{1}{3}}$

$x^3=-27$

$x^3+27=0$

$x^3+3^3=0$

$\because \quad (a^3+b^3)=(a+b)(a^2-ab+b^2)$

$(x+3)\left[x^2-(x)(3)+3^2\right]=0$

Either $(x+3)(x^2-3x+9)=0$

$x+3=0 \quad$ or $\quad x^2-3x+9=0$

Now we solve $x^2-3x+9=0$ by formula

$a=1,\ b=-3,\ c=19$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(19)}}{2\times1}$$

$$x=\frac{3\pm\sqrt{9-36}}{2}$$

$$x=\frac{3\pm\sqrt{-27}}{2}$$

$$x=\frac{3\pm\sqrt{(9)(-3)}}{2}$$

$$x=\frac{3\pm3\sqrt{-3}}{2}$$

$$x=-3\left(\frac{-1\pm\sqrt{-3}}{2}\right)$$

$$x=-3\left(\frac{-1+\sqrt{-3}}{2}\right) \quad \text{or} \quad x=-3\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$x=-3\omega \quad$ or $\quad x=-3\omega^2$

So cube roots of –27 are –3 , $-3\omega$ and $-3\omega^2$

(iv) **Cube roots of 64**

**Solution:**

Let $x=(64)^{\frac{1}{3}}$

$x^3=64$

$x^3-64=0$

$x^3-4^3=0$

$\because \quad a^3-b^3=(a-b)(a^2+ab+b^2)$

$(x-4)\left[x^2+(x)(4)+4^2\right]=0$

$(x-4)(x^2+4x+16)=0$

Either $x-4=0$ or $x^2+4x+16=0$

$x=4$

Now we solve $x^2+4x+16=0$ by formula

$a=1\ ,\ b=4\ ,\ c=16$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{4^2-4\times1\times16}}{2(1)}$$

$$x=\frac{-4\pm\sqrt{16-64}}{2}$$

$$x=\frac{-4\pm\sqrt{-48}}{2}$$

$$x=\frac{-4\pm\sqrt{16(-3)}}{2}$$

$$x=\frac{-4\pm4\sqrt{-3}}{2}$$

$$x=\frac{4\left(-1\pm\sqrt{-3}\right)}{2}$$

Either

$$x=4\left(\frac{-1+\sqrt{-3}}{2}\right) \quad \text{or} \quad x=4\left(\frac{-1-\sqrt{-3}}{2}\right)$$

Here $\omega=\frac{-1+\sqrt{-3}}{2}$ , $\omega^2=\frac{-1-\sqrt{-3}}{2}$

Therefore,

$x=4\omega$, or $\quad x=4\omega^2$

So cube roots of 64 are 4 , $4\omega$ *and* $4\omega^2$

**Q.2 Evaluate**

**(i)** $(1-\omega-\omega^2)^7$

**Solution:** $(1-\omega-\omega^2)^7$

$$=\left[1-\left(\omega+\omega^2\right)\right]^7$$

$\because 1+\omega+\omega^2=0$

$\omega+\omega^2=-1$

$$=\left[1-(-1)\right]^7$$
$$=(1+1)^7$$
$$=2^7$$
$$=128$$

**(ii)** $(1-3\omega-3\omega^2)^5$

**Solution:** $(1-3\omega-3\omega^2)^5$

$$=\left[1-3(\omega+\omega^2)\right]^5$$

$\because 1+\omega+\omega^2=0$

$\omega+\omega^2=-1$

$$=\left[1-3(-1)\right]^5$$
$$=(1+3)^5$$
$$=4^5=1024$$

**(iii)** $(9+4\omega+4\omega^2)^3$

**Solution:** $(9+4\omega+4\omega^2)^3$

$$=\left[9+4(\omega+\omega^2)\right]^3$$
$$=\left[9+4(-1)\right]^3 \quad (\because \omega+\omega^2=-1)$$
$$=\left[9-4\right]^3$$
$$=5^3$$
$$=125$$

**(iv)** $(2+2\omega-2\omega^2)(3-3\omega+3\omega^2)$

**Solution:** $(2+2\omega-2\omega^2)(3-3\omega+3\omega^2)$

$$=[2(1+\omega)-2\omega^2][3+3\omega^2-3\omega]$$
$$=[2(1+\omega)-2\omega^2][3(1+\omega^2)-3\omega]$$

$\because 1+\omega+\omega^2=0$

$1+\omega=-\omega^2 \qquad 1+\omega^2=-\omega$

$$\therefore \quad =[2(-\omega^2)-2\omega^2][3(-\omega)-3\omega]$$
$$=(-2\omega^2-2\omega^2)(-3\omega-3\omega)$$
$$=(-4\omega^2)(-6\omega)$$
$$=24\,\omega^3 \qquad \text{As } \omega^3=1$$
$$=24\,(1)$$
$$=24$$

**(v)** $\left(-1+\sqrt{-3}\right)^6+\left(-1-\sqrt{-3}\right)^6$

**Solution:**

$$\left(-1+\sqrt{-3}\right)^6+\left(-1-\sqrt{-3}\right)^6 \;.......(i)$$

As $\dfrac{-1+\sqrt{-3}}{2}=\omega$ *and* $\dfrac{-1-\sqrt{-3}}{2}=\omega^2$

$-1+\sqrt{-3}=2\omega$ *and* $-1-\sqrt{-3}=2\omega^2$

Now equation (i) becomes

$$=(2\omega)^6+(2\omega^2)^6$$
$$=2^6\,\omega^6+2^6\,\omega^{12}$$
$$=2^6\left[(\omega^3)^2+(\omega^3)^4\right] \qquad \text{As } \omega^3=1$$
$$=2^6\left[(1)^2+(1)^4\right]$$
$$=64\,(1+1)$$
$$=64\,(2)$$
$$=128$$

**(vi)** $\left(\dfrac{-1+\sqrt{-3}}{2}\right)^9+\left(\dfrac{-1-\sqrt{-3}}{2}\right)^9$

**Solution:**

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^9+\left(\frac{-1-\sqrt{-3}}{2}\right)^9 \;......(i)$$

As $\dfrac{-1+\sqrt{-3}}{2}=\omega$ *and* $\dfrac{-1-\sqrt{-3}}{2}=\omega^2$

Now equation (i) becomes

$$=(\omega)^9+(\omega^2)^9$$
$$=\omega^9+\omega^{18}$$
$$=(\omega^3)^3+(\omega^3)^6$$
$$=(1)^3+(1)^6 \qquad \text{As } \omega^3=1$$
$$=1+1$$
$$=2$$

**(vii)** $\omega^{37}+\omega^{38}-5$

**Solution:**

$$\omega^{37}+\omega^{38}-5$$
$$=\omega^{36}\omega+\omega^{36}\,\omega^2-5$$
$$=(\omega^3)^{12}\,\omega+(\omega^3)^{12}\,\omega^2-5$$
$$=(1)^{12}\,\omega+(1)^{12}\,\omega^2-5 \qquad \text{As } \omega^3=1$$
$$=1\,\omega+1\,\omega^2-5$$
$$=(\omega+\omega^2)-5$$
$$=(-1)-5 \qquad \text{As } 1+\omega+\omega^2=0$$
$$=-1-5=-6 \qquad \omega+\omega^2=-1$$

**(viii)** $\omega^{-13} + \omega^{-17}$

**Solution:**

$\omega^{-13} + \omega^{-17}$

$= \dfrac{1}{\omega^{13}} + \dfrac{1}{\omega^{17}}$

$= \dfrac{1}{\omega^{12}\omega} + \dfrac{1}{\omega^{15}\omega^{2}}$

$= \dfrac{1}{(\omega^3)^4\omega} + \dfrac{1}{(\omega^3)^5\omega^2}$

$= \dfrac{1}{(1)^4\omega} + \dfrac{1}{(1)^5\omega^2}$ $\quad \because \omega^3 = 1$

$= \dfrac{1}{\omega} + \dfrac{1}{\omega^2}$

$= \dfrac{\omega^2 + \omega}{(\omega)(\omega^2)}$

$= \dfrac{-1}{\omega^3}$ $\quad \because 1 + \omega + \omega^2 = 0$, $\omega + \omega^2 = -1$

$= \dfrac{-1}{1}$

$= -1$

**Q.3. Prove that,**

$x^3 + y^3 = (x+y)(x+\omega y)(x+\omega^2 y)$ 02(032)

**Solution:** Let,

R.H.S. $= (x+y)(x+\omega y)(x+\omega^2 y)$

$= (x+y)(x^2 + \omega^2 xy + \omega xy + \omega^3 y^2)$

$= (x+y)[x^2 + (\omega^2+\omega)xy + \omega^3 y^2]$

$\because 1+\omega+\omega^2 = 0 \Rightarrow \omega+\omega^2 = -1$ and $\omega^3 = 1$

$= (x+y)(x^2 + (-1)xy + 1y^2)$

$= (x+y)(x^2 - xy + y^2)$

Using Formula: $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$= x^3 + y^3 =$ L.H.S.

**Q.4. Prove that**

$$x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$$

**Solution:**

Let: R.H.S

$= (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$

$= (x+y+z)(x^2 + \omega^2 xy + \omega xz + \omega yx + \omega^3 y^2 + \omega^2 yz + \omega^2 xz + \omega^4 zy + \omega^3 z^2)$

$= (x+y+z)[(x^2 + \omega^3 y^2 + \omega^3 z^2 + (\omega^2+\omega)xy + (\omega^2+\omega^4)yz + (\omega+\omega^2)zx]$

$\because 1+\omega+\omega^2 = 0 \Rightarrow \omega+\omega^2 = -1$ and $\omega^3 = 1$

$= (x+y+z)\ [x^2 + 1y^2 + 1z^2 + (-1)xy + (\omega^2+\omega^3\omega)yz + (-1)zx]$

$= (x+y+z)\ [x^2 + 1y^2 + 1z^2 + (-1)xy + (\omega^2+1\omega)yz + (-1)zx]$

$= (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$= x^3+y^3+z^3-3xyz =$ L.H.S

Using Formula

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc$

**Q.5. Prove that** 02(034)

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$...........2n factors = 1

**Solution:**

Let L.H.S.

$=(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ ......... 2n factors

$=(1+\omega)(1+\omega^2)(1+\omega\omega^3)(1+\omega^6\omega^2)$ .... 2n factors

$\because \omega^3 = 1 \Rightarrow \omega^6 = (\omega^3)^2 = (1)^2 = 1$

$=(1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)$ .......... 2n factors

$=\left[(1+\omega)(1+\omega^2)\right]\left[(1+\omega)(1+\omega^2)\right]$..........n factors

$=\left[(1+\omega)(1+\omega^2)\right]^n$

$=\left[1+\omega^2+\omega+\omega^3\right]^n$

$=\left[1+\omega+\omega^2+\omega^3\right]^n$ $\quad \because 1+\omega+\omega^2 = 0$, $\omega^3 = 1$

$=[0+1]^n$

$=[1]^n$

$= 1 =$ R. H. S $\quad \Rightarrow$ L.H.S = R.H.S