EXERCISE 2.4

Q.1 If α, β are the roots of the equations $x^2 + px + q = 0$ then evaluate

(i) $\alpha^2+\beta^2$ (ii) $\alpha^3\beta+\alpha\beta^3$ (iii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

Solution:

$a = 1$, $b = p$, $c = q$

Sum of roots,

$$\alpha+\beta = \frac{-b}{a} = \frac{-p}{1} = -p$$

Product of roots

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

(i) $\alpha^2+\beta^2$

As

$$(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$$

or $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

$$= (\alpha+\beta)^2-2\alpha\beta$$

$$= (-p)^2-2(q)$$

$$= p^2-2q$$

(ii) $\alpha^3\beta+\alpha\beta^3$

$$= \alpha\beta(\alpha^2+\beta^2)$$

$$= \alpha\beta\left[(\alpha+\beta)^2-2\alpha\beta\right]$$

$$= q\left[(-p)^2-2q\right]$$

$$= q(p^2-2q)$$

(iii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$$= \frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$= \frac{1}{\alpha\beta}(\alpha^2+\beta^2)$$

$$= \frac{1}{\alpha\beta}\left[(\alpha+\beta)^2-2\alpha\beta\right]$$

$$= \frac{1}{q}\left[(-p)^2-2\times q\right]$$

$$= \frac{1}{q}(p^2-2q)$$

Q.2 If α, β are the roots of the equation $4x^2-5x+6=0$, the find the value of

(i) $\frac{1}{\alpha}+\frac{1}{\beta}$ (ii) $\alpha^2\beta^2$ (iii) $\frac{1}{\alpha^2\beta}+\frac{1}{\alpha\beta^2}$ (iv) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Solution: $4x^2-5x+6=0$

$ax^2+bx+c=0$

$a=4,\ b=-5,\ c=6$

Sum of roots,

$$\alpha+\beta=\frac{-b}{a}=-\frac{(-5)}{4}=\frac{5}{4}$$

Product of roots,

$$\alpha\beta=\frac{c}{a}=\frac{6}{4}$$

(i) $$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{5/4}{6/4}=5/6$$

(ii) $$\alpha^2\beta^2=(\alpha\beta)^2=\left(\frac{6}{4}\right)^2=\left(\frac{3}{2}\right)^2=\frac{9}{4}$$

(iii) $$\frac{1}{\alpha^2\beta}+\frac{1}{\alpha\beta^2}=\frac{\beta+\alpha}{\alpha^2\beta^2}=\frac{\alpha+\beta}{(\alpha\beta)^2}$$

$$=\frac{\frac{5}{4}}{\left(\frac{6}{4}\right)^2}=\frac{\frac{5}{4}}{\frac{36}{16}}=\frac{5}{4}\times\frac{16}{36}=\frac{5}{9}$$

(iv) $$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

Using Formula: $(\alpha+\beta)^3=\alpha^3+\beta^3+3(\alpha\beta)(\alpha+\beta)$

$\Rightarrow\ \alpha^3+\beta^3=(\alpha+\beta)^3-3(\alpha\beta)(\alpha+\beta)$

$$=\frac{(\alpha+\beta)^3-3(\alpha\beta)(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{\left(\frac{5}{4}\right)^3-3\left(\frac{6}{4}\right)\left(\frac{5}{4}\right)}{\left(\frac{6}{4}\right)}=\left(\frac{125}{64}-\frac{90}{16}\right)\frac{4}{6}$$

$$=\left(\frac{125-360}{64}\right)\frac{4}{6}=\frac{-235}{96}$$

Q.3 If α, β are the roots of the equation $\ell x^2+mx+n=0$ $(\ell\neq 0)$ then find the value of

(i) $\alpha^3\beta^2+\alpha^2\beta^3$ (ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Solution: $lx^2+mx+n=0$

$ax^2+bx+c=0$

$a=l,\ b=m,\ c=n$

If α,β be the Roots of given equation

Sum of roots $=\frac{-b}{a}$

$$\alpha+\beta=\frac{-m}{l}$$

Product of Roots $=\frac{c}{a}$

$$\alpha\beta=\frac{n}{l}$$

(i) $\alpha^3\beta^2+\alpha^2\beta^3$

$=\alpha^2\beta^2(\alpha+\beta)$

$=(\alpha\beta)^2(\alpha+\beta)$

$=\left(\frac{n}{l}\right)^2\left(\frac{-m}{l}\right)$

$=\left(\frac{n^2}{l^2}\right)\left(\frac{-m}{l}\right)$

$=\frac{-mn^2}{l^3}$

(ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$=\frac{\left(\frac{-m}{l}\right)^2-2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2}=\frac{\frac{m^2}{l^2}-\frac{2n}{l}}{\frac{n^2}{l^2}}$

$=\left(\frac{m^2-2nl}{l^2}\right)\cdot\frac{l^2}{n^2}$

$=\left(\frac{m^2-2nl}{n^2}\right)$

$=\frac{1}{n^2}\left(m^2-2nl\right)$