EXERCISE 2.5

Q.1 Write the quadratic equation having following roots.

(a) 1, 5 (b) 4, 9 (c) −2, 3
(d) 0,−3 (e) 2,−6 (f) −1, −7
(g) 1+i, 1−i (h) $3+\sqrt{2}$, $3-\sqrt{2}$

(a) 1, 5

Solution: Since 1 and 5 are the roots of the required quadratic equation, therefore

Sum of roots = S = 1 + 5 = 6

Product of roots = P = 1×5 = 5

As $x^2 - Sx + P = 0$ so the required equation is

$$x^2 - 6x + 5 = 0$$

(b) 4, 9

Solution: Since 4 and 9 are the roots of the required quadratic equation, therefore

Sum of roots = S = $4+9=13$

Product of roots = P = $4\times 9 = 36$

As $x^2 - Sx + P = 0$

So the required equation is

$$x^2 - 13x + 36 = 0$$

(c) −2, 3

Since −2, 3 are the roots of required quadratic equation, therefore

Sum of roots = S = −2 + 3 = 1

Product of roots = P = −2 ×3 = −6

As $x^2 - Sx + P = 0$

Therefore the required quadratic equation is

$$x^2 - x - 6 = 0$$

(d) 0, −3

Since 0, −3 are the roots of required quadratic equation therefore

Sum of roots = S = 0 + (−3) = −3

Product of roots = P = $0\times(-3) = 0$

As $x^2 - Sx + P = 0$

Therefore the required quadratic equation is

$$x^2 + 3x + 0 = 0 \Rightarrow$$

$$\Rightarrow \quad x^2 + 3x = 0$$

(e) 2, −6

Solution: Since 2 and −6 are the roots of the required quadratic equation therefore

Sum of roots = S = 2 + (−6) = 2 − 6 = − 4

Product of roots = P = 2 × (−6) = −12

As $x^2 - Sx + P = 0$ so the required equation is

$$x^2 + 4x - 12 = 0$$

(f) −1, −7

Solution: Since −1 and −7 are the roots of the required quadratic equation therefore

Sum of roots = S = (−1) + (−7)

$= -1 - 7 = -8$

Product of roots = P = $(-1)(-7) = 7$

As $x^2 - Sx + P = 0$ so the required equation is

$$x^2 + 8x + 7 = 0$$

(g) $1+i$, $1-i$

Solution: Since 1 + i and 1 − i are the roots of the required quadratic equation therefore

Sum of roots = S = $1+\not{i}+1-\not{i}=2$

Product of roots = P = $(1+i)(1-i)$

$$P = (1)^2 - (i)^2$$

$$P = 1 - (-1)$$

$$P = 1 + 1 = 2$$

As $x^2 - Sx + P = 0$ so the required equation is

$$x^2 - 2x + 2 = 0$$

(h) $3+\sqrt{2}, 3-\sqrt{2}$

Solution: $3+\sqrt{2}$ and $3-\sqrt{2}$ are the roots of the required quadratic equation therefore

Sum of roots = S = $3+\sqrt{2}+3-\sqrt{2}=6$

Product of roots

$$P = \left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$$

$$P = (3)^2 - \left(\sqrt{2}\right)^2$$

$$P = 9 - 2 = 7$$

As $x^2 - Sx + P = 0$, so the required equation is

$$x^2 - 6x + 7 = 0$$

Q.2 If α, β are the roots of the equation $x^2 - 3x + 6 = 0$. Form equation whose roots are (a) $2\alpha+1, 2\beta+1$ (b) α^2, β^2

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$ (d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

(e) $\alpha+\beta, \frac{1}{\alpha}+\frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

a = 1, b = −3, c = 6

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta=6}$$

(a) $2\alpha+1, 2\beta+1$

Sum of roots

$S = 2\alpha+1+2\beta+1$

$S = 2\alpha+2\beta+2$

$S = 2(\alpha+\beta)+2$

$S = 2(3)+2 = 6+2 = 8$

$\boxed{S=8}$

Product of roots

$P = (2\alpha+1)(2\beta+1)$

$P = 4\alpha\beta+2\alpha+2\beta+1$

$P = 4\alpha\beta+2(\alpha+\beta)+1$

$P = 4(6)+2(3)+1$

$P = 24+6+1 = 31$

$\boxed{P=31}$

Using $x^2 - Sx + P = 0$, we have

$x^2 - 8x + 31 = 0$

(b) α^2, β^2

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

a = 1, b = −3, c = 6

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta=6}$$

Sum of roots = $S = \alpha^2+\beta^2$

$S = (\alpha+\beta)^2 - 2\alpha\beta$

$S = (3)^2 - 2(6)$

$S = 9 - 12 = -3$

$\boxed{S=-3}$

Product of roots = $P = \alpha^2\beta^2$

$P = (\alpha\beta)^2$

$P = (6)^2 = 36$

$\boxed{P=36}$

Using $x^2 - Sx + P = 0$, we have

$x^2 + 3x + 36 = 0$

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

a = 1, b = −3, c = 6

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta=6}$$

Sum of roots = $S = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$S = (\alpha+\beta).\frac{1}{\alpha\beta}$

$S = 3\times\frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$\boxed{S=\frac{1}{2}}$

Product of roots = $P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$

$P = \frac{1}{\alpha\beta} = \frac{1}{6}$ $\boxed{P=\frac{1}{6}}$

Using $x^2 - Sx + P = 0$, we have

$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$

Multiplying by '6' on both side, we have

$6x^2 - 3x + 1 = 0$

(d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \ b = -3, \ c = 6$

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta=6}$$

Sum of roots = S $= \frac{\alpha}{\beta} + \frac{\beta}{a}$

$$S = \frac{\alpha^2+\beta^2}{\beta\alpha}$$

$$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(3)^2 - 2(6)}{6}$$

$$S = \frac{9-12}{6}$$

$$S = \frac{-3}{6}$$

$$\boxed{S = \frac{-1}{2}}$$

Product of roots = P = $\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$

Using $x^2 - Sx + P = 0$, we have

$$x^2 + \frac{1}{2}x + 1 = 0$$

Multiplying both sides by '2', we have

$$2x^2 + x + 2 = 0$$

(e) $\alpha+\beta, \frac{1}{\alpha} + \frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \ b = -3, \ c = 6$

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta=6}$$

Sum of roots = S $= (\alpha+\beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$S = (\alpha+\beta) + \frac{\beta+\alpha}{\alpha\beta}$$

$$S = (\alpha+\beta) + \frac{(\alpha+\beta)}{a\beta}$$

$$S = 3 + \frac{3}{6}$$

$$S = 3 + \frac{1}{2}$$

$$S = \frac{6+1}{2}$$

$$\boxed{S = \frac{7}{2}}$$

Product of roots = P $= (\alpha+\beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$P = (\alpha+\beta)\left(\frac{\beta+\alpha}{\alpha\beta}\right)$$

$$P = (\alpha+\beta)\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$P = 3\left(\frac{3}{6}\right)$$

$$\boxed{P = \frac{3}{2}}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying both sides by '2' we have

$$2x^2 - 7x + 3 = 0$$

Q.3 If α, β are the roots of the equation $x^2+px+q=0$ From equation whose roots are

(a). α^2, β^2 (b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

Since α, β are the roots of the equation

$x^2+px+q=0$

$ax^2+bx+c=0$

By comparing the coefficients of these equations, we have

$$a=1,\ b=p,\ c=q$$

$$\alpha+\beta=\frac{-b}{a}=\frac{-p}{1}=-p \Rightarrow \boxed{\alpha+\beta=-p}$$

$$\alpha\beta=\frac{c}{a}=\frac{q}{1}=q \Rightarrow \boxed{\alpha\beta=q}$$

(a) α^2, β^2

Sum of roots $= S = \alpha^2+\beta^2$

$$S=(\alpha+\beta)^2-2\alpha\beta$$

$$S=(-p)^2-2q$$

$$S=p^2-2q$$

Product of roots $= P = \alpha^2\beta^2$

$$P=(\alpha\beta)^2$$

$$P=q^2$$

Using $x^2-Sx+P=0$, we have

$$x^2-(p^2-2q)x+q^2=0$$

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

Since α, β are the roots of the equation

$x^2+px+q=0$

$ax^2+bx+c=0$

By comparing the coefficients of these equations, we have

$$a=1,\ b=p,\ c=q$$

$$\alpha+\beta=\frac{-b}{a}=\frac{-p}{1}=-p \Rightarrow \boxed{\alpha+\beta=-p}$$

$$\alpha\beta=\frac{c}{a}=\frac{q}{1}=q \Rightarrow \boxed{\alpha\beta=q}$$

Sum of roots $= S = \frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$$S=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$S=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$S=\frac{(-p)^2-2(q)}{q}$$

$$S=\frac{p^2-2q}{q}$$

Product of roots $= P = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)=1$

Using $x^2-Sx+P=0$, we have

$$x^2-\left(\frac{p^2-2q}{q}\right)x+1=0$$

Multiplying by q

$$qx^2-(p^2-2q)x+q=0$$