EXERCISE 2.6

Q.1 Use synthetic division to find the quotient and the remainder, when

(i) $(x^2+7x-1)\div(x+1)$

As $x+1=x-(-1)$ So $a=-1$

Now write the co–efficient of dividend in a row and $a=-1$ on the left side

	1	7	–1
–1	↓	–1	–6
	1	6	–7

Quotient $Q(x)=x+6$

Remainder R = –7

(ii) $(4x^3-5x+15)\div(x+3)$

$(4x^3+0x^2-5x+15)\div(x+3)$

or As $x+3=x-(-3)$, So $a=-3$

Now write the co–efficient of dividend in a row and $a=-3$ on the left side

	4	0	– 5	15
–3	↓	–12	36	–93
	4	–12	31	–78

Quotient = $Q(x)=4x^2-12x+31$

Remainder = $R=-78$

(iii) $(x^3+x^2-3x+2)\div(x-2)$

As $(x-2)$ So $a=2$

Now write the co–efficient of dividend in a row and $a=2$ on the left side.

	1	1	– 3	2
2	↓	2	6	6
	1	3	3	8

Quotient $Q(x)=x^2+3x+3$

Remainder R = 8

Q.2 Find the value of h using synthetic division, if 3 is the zero of the polynomial $2x^3–3hx^2+9$

(i) Solution: Let $P(x)=2x^3-3hx^2+0x+9$ and its zero is 3. Then by synthetic division.

	2	$-3h$	0	9
3	↓	6	$18-9h$	$54-27h$
	2	$6-3h$	$18-9h$	$63-27h$

Remainder = 63 – 27h

Since 3 is the zero of the polynomial, therefore Remainder = 0

$63-27h=0$

$63=27h$

$\Rightarrow h=\frac{63}{27}$ $\Rightarrow \boxed{h=\frac{7}{3}}$

(ii) Find the value of h using synthetic division, if 1 is the zero of the polynomial $x^3–2hx^2+11$

Solution:

Let $P(x)=x^3-2hx^2+0x+11$ and its zero is 1. Then by synthetic division

	1	$-2h$	0	11
1	↓	1	$1-2h$	$1-2h$
	1	$1-2h$	$1-2h$	$12-2h$

Remainder = 12 – 2h

Since 1 is the zero of the polynomial

So, Remainder = 0 that is

$12-2h=0$

$12=2h$

$\Rightarrow h=\frac{12}{2}$

$\boxed{h=6}$

(iii) Find the value of h using synthetic division, if –1 is the zero of the Polynomial $2x^3+5hx-23$

Solution:

Let $P(x)=2x^3+5hx-23$

$P(x)=2x^3+0x^2+5hx-23$

If–1 is zero of p(x) then by Synthetic division

	2	0	5h	−23
−1	↓	−2	2	−5h − 2
	2	−2	5h+2	−5h − 25

Remainder = $-5h-25$

Since –1 is the zero of the Polynomial

So Remainder = 0

$-5h-25=0$

$-5h=25$

$h=\frac{25}{-5}$ ⇒ $\boxed{h=-5}$

Q.3 Use synthetic division to find the values of l and m,

(i). if (x+3) and (x–2) are the factors of the polynomial $x^3+4x^2+2lx+m$

Solution: Since $(x+3)$ and $(x-2)$ are the factors of $P(x)=x^3+4x^2+2\ell x+m$

Therefore –3 and 2 are the zeros of polynomial P(x). Now by synthetic division.

	1	4	2ℓ	m
−3	↓	−3	−3	$-6\ell+9$
	1	1	$2\ell-3$	$m+(-6\ell+9)$

Since –3 is the zero of polynomial, therefore remainder is zero that is

$m-6\ell+9=0$

⇒ $m-6\ell=-9$ ....(i)

And

	1	4	2ℓ	m
2	↓	2	12	$4\ell+24$
	1	6	$2\ell+12$	$m+4\ell+24$

Since 2 is the zero of polynomial, therefore remainder is zero that is

$m+4\ell+24=0$

$m+4\ell=-24$ ....(ii)

Subtracting equations (ii) from (i)

$$\begin{aligned} m-6\ell&=-9\\ \pm m\pm 4\ell&=\mp 24\\ \hline -10\ell&=15 \end{aligned}$$

$\ell=\frac{15}{-10}=\frac{3}{-2}$

⇒ $\boxed{\ell=-\frac{3}{2}}$

Put it in equations (i), we get

$m-6\left(\frac{-3}{2}\right)=-9$

$m+\frac{18}{2}=-9$

$m+9=-9$

$m=-9-9$

$\boxed{m=-18}$

(ii). Find the values of l and m if (x–1) and (x+1) are the factors of the polynomial $x^3-3lx^2+2mx+6$

Solution: Since $(x-1)$ and $(x+1)$ are the factors of $P(x)=x^3-3\ell x^2+2mx+6$

Therefore 1 and –1 are zeros of polynomial P(x). Now by synthetic division

	1	-3ℓ	$2m$	6
1	↓	1	$1-3\ell$	$1-3\ell+2m$
	1	$1-3\ell$	$1-3\ell+2m$	$7-3\ell+2m$

Since 1 is the zero of polynomial, therefore remainder is zero that is

$7-3\ell+2m=0$

$2m-3\ell=-7$ .... (i)

And

	1	-3ℓ	$2m$	6
−1	↓	−1	$1+3\ell$	$-1-3\ell-2m$
	1	$-1-3\ell$	$1+3\ell+2m$	$5-3\ell-2m$

Since –1 is the zero of polynomial therefore remainder is zero that is

$$5-3\ell-2m=0$$

$\Rightarrow \quad 2m+3\ell=5 \;...........(ii)$

Adding equations (i) and (ii)

$$2m-3\ell=-7$$

$$2m+3\ell=5$$

$$4m=-2$$

$$m=\frac{-2}{4}$$

$$\boxed{m=\frac{-1}{2}}$$

Put it in equation (i),

$$2\left(-\frac{1}{2}\right)-3\ell=-7$$

$$-1-3\ell=-7$$

$$-3\ell=-7+1$$

$$-3\ell=-6$$

$\Rightarrow \quad \ell=\frac{-6}{-3}$

$\Rightarrow \quad \boxed{\ell=2}$

Q.4 Solve by **using synthetic division,**

(i) If 2 is the root of the equation $x^3-28x+48=0$

Solution: Let $P(x) = x^3 + 0x^2 - 28x + 48$

Since 2 is the root of the equation $x^3-28x+48=0$ then by synthetic division.

	1	0	–28	48
2	↓	2	4	–48
	1	2	–24	0

The depressed equation is

$$x^2+2x-24=0$$

$$x^2+6x-4x-24=0$$

$$x(x+6)-4(x+6)=0$$

$$(x+6)(x-4)=0$$

Either $x+6=0$ or $x-4=0$

$x=-6$ or $x=4$

Thus 2, –6 and 4 are the roots of the given equation

(ii). If 3 is the root of the equation $2x^3-3x^2-11x+6=0$

Solution: Since 3 is the root of the equation

$$2x^3-3x^2-11x+6=0$$

Then by synthetic division

	2	–3	–11	6
3	↓	6	9	–6
	2	3	–2	0

The depressed equation is

$$2x^2+3x-2=0$$

$$2x^2+4x-x-2=0$$

$$2x(x+2)-1(x+2)=0$$

$$(x+2)(2x-1)=0$$

Either $x+2=0$ or $2x-1=0$

$\boxed{x=-2}$ or $2x=1$

$$\boxed{x=\frac{1}{2}}$$

Thus, 3, –2 and $\frac{1}{2}$ are the roots of the given equation.

(iii). If –1 is the root of the equation $4x^3-x^2-11x-6=0$

Solution: Since –1 is the root of the equation.

$$4x^3-x^2-11x-6=0$$

Then by synthetic division

	4	–1	–11	–6
–1	↓	–4	5	6
	4	–5	–6	0

The depressed equation is

$$4x^2-5x-6=0$$

$$4x^2-8x+3x-6=0$$

$$4x(x-2)+3(x-2)=0$$

$$(x-2)(4x+3)=0$$

Either $x-2=0$ or $4x+3=0$

$\boxed{x=2}$ or $4x=-3$

$$\boxed{x=\frac{-3}{4}}$$

Thus –1, 2 and $\frac{-3}{4}$ are the roots of the given equation

Q.5

(i) Solve by using synthetic division, if 1 and 3 are the roots of the equation
$x^4 - 10x^2 + 9 = 0$

Solution: Since 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

Then by synthetic division, we get

	1	0	−10	0	9
1	↓	1	1	−9	−9
	1	1	−9	−9	0
3	↓	3	12	9	
	1	4	3	0	

Thus the depressed equation is

$$x^2 + 4x + 3 = 0$$
$$x + 3x + x + 3 = 0$$
$$x(x+3) + 1(x+3) = 0$$
$$(x+3)(x+1) = 0$$

Either $x + 3 = 0$ or $x + 1 = 0$

$\boxed{x = -3}$ $\boxed{x = -1}$

Hence 1, 3, −3 and −1 are the roots of the given equation

(ii) Solve by using synthetic division, if 3 and −4 are the roots of the equation
$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$ **02(084)**

Solution: Since 3 and −4 are the roots of the given equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$ then by synthetic division, we get

	1	2	−13	−14	24
3	↓	3	15	6	−24
	1	5	2	−8	0
−4	↓	−4	−4	8	
	1	1	−2	0	

The depressed equation is

$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$
$$x(x+2) - 1(x+2) = 0$$
$$(x+2)(x-1) = 0$$

Either $x + 2 = 0$ or $x - 1 = 0$

$\boxed{x = -2}$ $\boxed{x = 1}$

Thus 3, −4, −2 and 1 are the roots of the given equation.