EXERCISE 2.8

Q. 1 The product of two positive consecutive numbers is 182. Find the numbers.

Solution:

Suppose first positive number = x

Very next positive number = x + 1

By given condition

$$x(x+1)=182$$
$$x^2+x-182=0$$
$$x^2+14x-13x-182=0$$
$$x(x+14)-13(x+14)=0$$
$$(x+14)(x-13)=0$$

Either $x+14=0$ or $x-13=0$

$x=-14$ or $x=13$

As x is positive number therefore we neglect the negative value, So x = 13

Thus first positive number = x = 13

Very next positive number = x + 1

= 13+1 = 14

Thus 13 and 14 are two required consecutive positive numbers.

Q. 2 The sum of the squares of three positive consecutive numbers is 77. Find them.

Solution:

Let $x,(x+1)$ and $(x+2)$ be the three consecutive positive number

By Give condition

$$x^2+(x+1)^2+(x+2)^2=77$$
$$x^2+[x^2+(1)^2+2(1)(x)]+[(x)^2+(2)^2+2(x)(2)]=77$$
$$x^2+x^2+1+2x+x^2+4+4x=77$$
$$3x^2+6x+5-77=0$$
$$3x^2+6x-72=0$$
$$3\left[x^2+2x-24\right]=0$$
$$\therefore x^2+2x-24=0 \qquad (\because 3\neq 0)$$
$$x^2+6x-4x-24=0$$
$$x(x+6)-4(x+6)=0$$
$$(x+6)(x-4)=0$$

Either $x+6=0$ or $x-4=0$

$x=-6$ or $x=4$

As x is a positive number therefore we neglect the negative value and we take positive value of x i.e. x = 4

1^{st} Number $= x=4$

2^{nd} Number $= x+1=4+1=5$

3^{rd} Number $= x+2=4+2=6$

Thus 4, 5 and 6 are three required positive numbers.

Q. 3 The sum of five times a number and the square of the number is 204. Find the number.

Solution:

Let required number = x

Five times the number = 5x

Square of the number = x^2

By given condition

$$x^2+5x=204$$
$$x^2+5x-204=0$$
$$x^2+17x-12x-204=0$$
$$x(x+17)-12(x+17)=0$$
$$(x+17)(x-12)=0$$

Either

$x+17=0$ or $x-12=0$

$\boxed{x=-17}$ or $\boxed{x=12}$

Thus required number is –17 or 12.

Q. 4 The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Solution:

Let required number = x

Five less than three times the number = 3x–5

One less than four times the number = 4x–1

By given condition

$$(3x-5)(4x-1)=7$$
$$12x^2-3x-20x+5-7=0$$
$$12x^2-23x-2=0$$
$$12x^2-24x+x-2=0$$
$$12x(x-2)+1(x-2)=0$$
$$(x-2)(12x+1)=0$$

$x-2=0$ or $12x+1=0$
$x=2$ or $12x=-1$
$x=2$ or $x=\frac{-1}{12}$

Thus required number is 2 or $\frac{-1}{12}$

Q. 5 The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Solution:

Let required number = x

Reciprocal of the number = $\frac{1}{x}$

By given condition

$$x-\frac{1}{x}=\frac{15}{4}$$
$$\frac{x^2-1}{x}=\frac{15}{4}$$
$$4\left(x^2-1\right)=15x$$
$$4x^2-4-15x=0$$
$$4x^2-15x-4=0$$
$$4x^2-16x+1x-4=0$$
$$4x(x-4)+1(x-4)=0$$
$$(x-4)(4x+1)=0$$

Either $x-4=0$ or $4x+1=0$
$x=0+4$ or $4x=-1$
$\boxed{x=4}$ or $\boxed{x=\frac{-1}{4}}$

Thus required numbers is 4 or $\frac{-1}{4}$.

Q. 6 The sum of the squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

Solution:

Let

Digits at unit's place of a number = x

Digit at ten's place of a number = y

Required number = 10y + x

By 1st condition

$$x^2+y^2=65 \quad \ldots\ldots\ldots\ldots\ldots(i)$$

By 2nd condition

$$10y+x=9(x+y)$$
$$10y+x=9x+9y$$
$$10y-9y=9x-x$$
$$y=8x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Put value of y in equation (i)

$$x^2+(8x)^2=65$$
$$x^2+64x^2=65$$
$$65x^2=65$$
$$x^2=1$$
$$\sqrt{x^2}=\pm\sqrt{1}$$
$$x=\pm1$$
$$x=1 \quad or \quad x=-1$$

As x is a digit at unit's place which is always positive therefore we neglect the negative value and take the positive value i.e. $x=1$

Put x = 1 in equation (ii)

$$y=8(1)$$
$$y=8$$

So, required number $=10y+x$
$=10(8)+1$
$=80+1$
$=81$

Q. 7 The sum of the co–ordinates of a point is 9 and sum of their squares is 45. Find the co–ordinates of the point.

Solution:

Let (x, y) are co–ordinates of required point.

By given conditions

$$x+y=9 \quad \ldots\ldots\ldots\ldots\ldots(i)$$
$$x^2+y^2=45 \quad \ldots\ldots\ldots\ldots(ii)$$

From equation (i)

$$x+y=9$$
$$x=9-y \quad \ldots\ldots\ldots\ldots\ldots(iii)$$

Putting this in equation (ii), we get

$$(9-y)^2+y^2=45$$
$$(9)^2-2(9)(y)+(y)^2+(y)^2=45$$
$$81-18y+y^2+y^2=45$$
$$2y^2-18y+81-45=0$$

$$2y^2 - 18y + 36 = 0$$
$$2(y^2 - 9y + 8) = 0$$
$$\therefore \quad y^2 - 9y^2 + 18 = 0 \qquad (\because 2 \neq 0)$$
$$y^2 - 6y - 3y + 18 = 0$$
$$y(y-6) - 3(y-6) = 0$$
$$(y-6)(y-3) = 0$$

Either $y - 6 = 0$ or $y - 3 = 0$

$y = 6$ or $y = 3$

Putting the values of y in equation (iii), we get

When y = 6	When y = 3
$x = 9 - 6$	x = 9 – 3
$x = 3$	x = 6

Thus co–ordinates of the point are either (3, 6) or (6, 3)

Q. 8 Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution: 02(105)

Suppose x and y are two integer

By given conditions

$$x + y = 9 \quad \ldots\ldots\ldots\ldots (i)$$
$$x^2 - y^2 = 9 \ldots\ldots\ldots\ldots (ii)$$

From equation (i)

$$x + y = 9$$
$$x = 9 - y \quad \ldots\ldots\ldots\ldots (iii)$$

Putting the value of x in equation (ii), we get

$$(9-y)^2 - y^2 = 9$$
$$(9)^2 + (y)^2 - 2(9)(y) - y^2 = 9$$
$$81 + \cancel{y^2} - 18y - \cancel{y^2} - 9 = 0$$
$$72 - 18y = 0$$
$$-18y = -72$$
$$y = \frac{-72}{-18}$$
$$y = 4$$

Putting the value of y in equation (iii), we get

$$x = 9 - y$$
$$x = 9 - 4$$
$$x = 5$$

So 4 and 5 are required integers.

Q. 9 Find two integers whose difference is 4 and whose squares differ by 72.

Solution:

Let x and y are two integers

By given conditions

$$x - y = 4 \quad \ldots\ldots\ldots\ldots (i)$$
$$x^2 - y^2 = 72 \quad \ldots\ldots\ldots\ldots (ii)$$

From equation (i)

$$x = 4 + y \quad \ldots\ldots\ldots\ldots (iii)$$

Putting the value of x in equation (ii), we get

$$(4+y)^2 - y^2 = 72$$
$$[(4)^2 + (y)^2 + 2(4)(y)] - y^2 = 72$$
$$16 + \cancel{y^2} + 8y - \cancel{y^2} = 72$$
$$16 + 8y = 72$$
$$8y = 72 - 16$$
$$8y = 56$$
$$y = \frac{56}{8}$$
$$y = 7$$

Putting the value of y in equation (iii)

$$x = 4 + y$$
$$x = 4 + 7$$
$$x = 11$$

So 7 and 11 are required integers

Q. 10 Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375cm².

Solution:

Let width of rectangle = x cm

Length of rectangle = y cm

Perimeter of rectangle = 80cm

Area of rectangle = 375 cm²

We know that

$$2(L + W) = P$$
$$2(x + y) = 80$$
$$x + y = \frac{80}{2}$$
$$x + y = 40 \quad \ldots\ldots\ldots\ldots (i)$$

Area = Length × Width

$$375 = x \times y$$
$$\Rightarrow \quad xy = 375 \quad \ldots\ldots\ldots\ldots (ii)$$

From equation (i)

$$x + y = 40$$
$$y = 40 - x$$

Put it in equation (ii)

$$x(40-x)=375$$
$$40x-x^2=375$$
$$0=x^2-40x+375$$
$$\Rightarrow \quad x^2-40x+375=0$$
$$x^2-25x-15x+375=0$$
$$x(x-25)-15(x-25)=0$$
$$(x-25)(x-15)=0$$

Either $x-25=0$ or $x-15=0$

$x=25$ or $x=15$

Putting these values of x in equation (i), we get

When $x=25$	When $x=15$
Then $25+y=40$	Then $15+y=40$
$y=40-25$	$y=40-15$
$y=15$	$y=25$
If x = 25 then y = 15	If x = 15 then y = 25

Thus dimensions of rectangle is either 25cm by 15cm or 15cm by 25cm.