EXERCISE 4.1

Resolve into partial fractions.

Q.1 $\dfrac{7x-9}{(x+1)(x-3)}$

Solution: $\dfrac{7x-9}{(x+1)(x-3)}$

Let $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{A}{x+1}+\dfrac{B}{x-3}$ ...... (i)

Multiplying equation (i) by (x + 1) (x – 3)

$7x-9 = A(x-3)+B(x+1)$ ...... (ii)

As equation (ii) is an identity which is true for all values of x.

Put $x-3=0$ i.e $x=3$ and

Put $x+1=0$ i.e $x=-1$

Putting $x = 3$ and $x=-1$ in (ii) we get

For $x = 3$

$7(3)-9 = +B(3+1)$

$21-9 = 4B$

$12 = 4B$

$\Rightarrow \boxed{B=3}$

For $x=-1$

$7(-1)-9 = A(-1-3)$

$-7-9 = -4A$

$-16 = -4A$

$\Rightarrow \boxed{A=4}$

Putting the value of A and B in equation (i)

We get the required partial fractions as.

$\dfrac{4}{x+1}+\dfrac{3}{x-3}$

Thus $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{4}{x+1}+\dfrac{3}{x-3}$

Q.2 $\dfrac{x-11}{(x-4)(x+3)}$

Solution: $\dfrac{x-11}{(x-4)(x+3)}$

Let $\dfrac{x-11}{(x-4)(x+3)} = \dfrac{A}{x-4}+\dfrac{B}{x+3}$ .....(i)

Multiplying by (x–4)(x+3) on both sides, we get

$x-11 = A(x+3)+B(x-4)$ .... (ii)

As equation (ii) is an identity which is true for all value of x.

Putting $x+3=0$ i.e $x=-3$

and $x-4=0$ i.e $x = 4$ in (ii) we get

For $x=-3$

$-3-11 = B(-3-4)$

$-14 = -7B$

$\Rightarrow \boxed{B=2}$

For $x=4$

$4-11 = A(4+3)$

$-7 = 7A$

$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$\dfrac{x-11}{(x-4)(x+3)} = \dfrac{-1}{x-4}+\dfrac{2}{x+3}$

Q.3 $\dfrac{3x-1}{x^2-1}$

Solution: $\dfrac{3x-1}{x^2-1}$

$\dfrac{3x-1}{x^2-1} = \dfrac{3x-1}{(x-1)(x+1)}$

Let $\dfrac{3x-1}{(x-1)(x+1)} = \dfrac{A}{x-1}+\dfrac{B}{x+1}$ ......(i)

Multiplying both sides by (x–1)(x+1), we get

$3x-1 = A(x+1)+B(x-1)$ .... (ii)

As equation (ii) is an identity which is true for all values of x.

Let $x+1 = 0$ i.e $x=-1$ and

$x-1=0$ i.e $x=1$

Putting $x=-1$ and $x=1$ in (ii) We get

For $x=1$

$3(1)-1 = A(1+1)$

$3-1 = 2A$

$2 = 2A$

$\Rightarrow \boxed{A=1}$

For $x=-1$

$3(-1)-1 = B(-1-1)$

$-3-1 = -2B$

$-4 = -2B$

$\Rightarrow \boxed{B=2}$

Hence the required partial fractions are

$\dfrac{3x-1}{(x-1)(x+1)} = \dfrac{1}{x-1}+\dfrac{2}{x+1}$

Q.4 $\dfrac{x-5}{x^2+2x-3}$

Solution: $\dfrac{x-5}{x^2+2x-3}$

$$\frac{x-5}{x^2+2x-3} = \frac{x-5}{x^2+3x-x-3}$$

$$= \frac{x-5}{x(x+3)-1(x+3)}$$

$$= \frac{x-5}{(x-1)(x+3)}$$

$$\frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1}+\frac{B}{x+3} \;\ldots\ldots\; \text{(i)}$$

Multiplying both sides by (x–1) (x+3), we get

$$x-5 = A(x+3)+B(x-1) \ldots\ldots\; \text{(ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x+3=0 \Rightarrow x=-3$

and $x-1=0 \Rightarrow x=1$

Putting $x=-3$ and $x=1$ in equation (ii) we get

For $x=-3$	For $x=1$
$-3-5 = +B(-3-1)$	$1-5 = A(1+3)$
$-8 = -4B$	$-4 = 4A$
$B=\dfrac{-8}{-4}$	$A=\dfrac{-4}{4}$
$\Rightarrow \boxed{B=2}$	$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$$\frac{x-5}{x^2+2x-3} = \frac{-1}{x-1}+\frac{2}{x+3}$$

Q.5 $\dfrac{3x+3}{(x-1)(x+2)}$

Solution: $\dfrac{3x+3}{(x-1)(x+2)}$

Let $\dfrac{3x+3}{(x-1)(x+2)} = \dfrac{A}{x-1}+\dfrac{B}{x+2} \;\ldots\ldots\; \text{(i)}$

Multiplying both sides by (x–1)(x+2), we get

$$3x+3 = A(x+2)+B(x-1) \ldots\ldots\; \text{(ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x-1=0$ i.e $x=1$

and $x+2=0$ i.e $x=-2$

Putting $x=1$ and $x=-2$ in equation (ii) we get

For $x=1$	For $x=-2$
$3(1)+3 = A(1+2)$	$3(-2)+3 = B(-2-1)$
$3+3 = 3A$	$-6+3 = -3B$
$6 = 3A$	$-3 = -3B$
$A=\dfrac{6}{3}$	$B=\dfrac{-3}{-3}$
$\Rightarrow \boxed{A=2}$	$\Rightarrow \boxed{B=1}$

Hence the required partial fractions are

$$\frac{3x+3}{(x-1)(x+2)} = \frac{2}{x-1}+\frac{1}{x+2}$$

Q.6 $\dfrac{7x-25}{(x-4)(x-3)}$

Solution: $\dfrac{7x-25}{(x-4)(x-3)}$

Let $\dfrac{7x-25}{(x-4)(x-3)} = \dfrac{A}{x-4}+\dfrac{B}{x-3}$

Multiplying both sides by(x–4) (x – 3), we get

$$7x-25 = A(x-3)+B(x-4) \ldots\ldots\; \text{(ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x-3=0$ i.e $x=3$

and $x-4=0$ i.e $x=4$

Putting $x=3$ and $x=4$ in equation (ii) we get

For $x=3$	For $x=4$
$7(3)-25 = B(3-4)$	$7(4)-25 = A(4-3)$
$21-25 = -B$	$28-25 = 1A$
$-4 = -B$	$3 = A$
$\Rightarrow \boxed{B=4}$	$\Rightarrow \boxed{A=3}$

Hence the required partial fractions are

$$\frac{7x-25}{(x-4)(x-3)} = \frac{3}{x-4}+\frac{4}{x-3}$$

Q.7 $\dfrac{x^2+2x+1}{(x-2)(x+3)}$

Solution: $\dfrac{x^2+2x+1}{(x-2)(x+3)}$ is an improper fraction. First we resolve it into proper fraction.

By long division we get

$$\begin{array}{r} 1 \\ x^2+x-6\overline{)x^2+2x+1} \\ \pm x^2 \pm x \mp 6 \\ \hline x+7 \end{array}$$

We have $\dfrac{x^2+2x+1}{x^2+x-6} = 1+\dfrac{x+7}{x^2+x-6}$

Let $\dfrac{x+7}{(x-2)(x+3)} = \dfrac{A}{x-2}+\dfrac{B}{x+3}$ ....(i)

Multiplying both sides by (x–2) (x +3), we get

$x + 7= A\ (x +3) + B(x - 2)$......... (ii)

As equation (ii) is an identity which is true for all values of x.

Let $x + 3 = 0$ i.e $x = -3$

and $x - 2 = 0$ i.e $x = 2$

Putting $x = -3$ and $x = 2$ in equation (ii) we get

For $x = -3$	For $x = 2$
$-3+7 = B(-3-2)$	$2+7 = A\ (2+3)$
$4 = -5B$	$9 = 5A$
$\Rightarrow \boxed{B = -\dfrac{4}{5}}$	$\Rightarrow \boxed{A = \dfrac{9}{5}}$

Hence the required partial fractions are

$$\frac{x^2+2x+1}{(x-2)(x+3)} = 1+\frac{9}{5(x-2)}-\frac{4}{5(x+3)}$$

Q.8 $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$

Solution: $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$ is an improper fraction.

First we resolve it into proper fraction.

$$\begin{array}{r} 2x+3 \\ 3x^2-2x-1\overline{)6x^3+5x^2-7} \\ \pm 6x^3 \mp 4x^2 \mp 2x \\ \hline 9x^2+2x-7 \\ \pm 9x^2 \mp 6x \mp 3 \\ \hline 8x-4 \end{array}$$

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = (2x+3)+\frac{8x-4}{(3x+1)(x-1)}$$

Now, Let $\dfrac{8x-4}{(3x+1)(x-1)} = \dfrac{A}{3x+1}+\dfrac{B}{x-1}$ ....(i)

Multiplying both sides by(3x+1) (x–1), we get

$8x - 4 = A\ (x - 1) + B(3x + 1)$...... (ii)

As equation (ii) is an identity which is true for all values of x.

Let $x - 1 = 0$ i.e $x = 1$

and $3x + 1 = 0$ i.e $x = -\dfrac{1}{3}$

Putting $x = 1$ and $x = \dfrac{-1}{3}$ in equation (ii) we get

For $x = 1$	For $x = \dfrac{-1}{3}$
$8(1) - 4 = B\,[3(1) + 1]$	$8\left(\dfrac{-1}{3}\right)-4 = A\left(\dfrac{-1}{3}-1\right)$
$-4 = 4B$	$\dfrac{-8}{3}-4 = \dfrac{A(-1-3)}{3}$
$4 = 4B$	$\dfrac{-8-12}{3} = \dfrac{A(-4)}{3}$
$\Rightarrow 4B = 4$	
$B = \dfrac{4}{4}$	$\dfrac{-20}{3} = \dfrac{-4}{3}A$
$\Rightarrow \boxed{B = 1}$	$\Rightarrow \boxed{A = 5}$

Hence the required partial functions are

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{5}{3x+1}+\frac{1}{x-1}$$