EXERCISE 4.3

Resolve into partial fractions.

Q.1 $\dfrac{3x-11}{(x+3)(x^2+1)}$

Solution: $\dfrac{3x-11}{(x+3)(x^2+1)}$

Let $\dfrac{3x-11}{(x+3)(x^2+1)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1}$ ... (i)

Multiplying both sides $(x+3)(x^2+1)$, we get

$3x - 11 = A(x^2+1) + (Bx+C)(x+3)$... (ii)

$3x - 11 = A(x^2+1) + Bx(x+3) + C(x+3)$... (iii)

Putting $x + 3 = 0$ i.e $x = -3$ in (ii), we get

$$3(-3) - 11 = A[(-3)^2 + 1]$$

$$-9 - 11 = A(9+1)$$

$$-20 = 10A$$

$$A = \frac{-20}{10}$$

$\Rightarrow$ $\boxed{A = -2}$

Now equating the coefficients of x^2 and x we get from equation (iii)

$A + B = 0$

$-2 + B = 0$

$B = 2$

$\Rightarrow$ $\boxed{B = 2}$

$3B + C = 3$

$3(2) + C = 3$

$6 + C = 3$

$C = 3 - 6$

$\Rightarrow$ $\boxed{C = -3}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{x+3} + \frac{2x-3}{x^2+1}$$

Q.2 $\dfrac{3x+7}{(x^2+1)(x+3)}$

Solution: $\dfrac{3x+7}{(x^2+1)(x+3)}$

Let $\dfrac{3x+7}{(x^2+1)(x+3)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x+3}$ ......(i)

Multiplying both sides by $(x^2+1)(x+3)$

$3x + 7 = (Ax + B)(x+3) + C(x^2+1)$

$3x+7 = Ax(x+3) + B(x+3) + C(x^2+1)$ .....(ii)

Putting $x + 3 = 0$ i.e $x = -3$ in (ii), we get

$$3(-3)+7 = C[(-3)^2+1]$$

$$-9+7 = C(9+1)$$

$$-2 = 10C$$

$$\Rightarrow \quad C = \frac{-2}{10}$$

$$\boxed{C = \frac{-1}{5}}$$

Now equating the coefficients of x^2 and x in equation (iii) we get

$A + C = 0$

$A + \left(\dfrac{-1}{5}\right) = 0$

$A - \dfrac{1}{5} = 0$

$\Rightarrow$ $\boxed{A = \dfrac{1}{5}}$

$3A + B = 3$

$3\left(\dfrac{1}{5}\right) + B = 3$

$B = 3 - \dfrac{3}{5}$

$B = \dfrac{15-3}{5}$

$\Rightarrow$ $\boxed{B = \dfrac{12}{5}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x+7}{(x^2+1)(x+3)} = \frac{x+12}{5(x^2+1)} - \frac{1}{5(x+3)}$$

Q.3 $\dfrac{1}{(x+1)(x^2+1)}$

Solution: $\dfrac{1}{(x+1)(x^2+1)}$

Let $\dfrac{1}{(x+1)(x^2+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}$ ....(i)

Multiplying both sides by (x+1) (x^2+1), we get

$$1 = A(x^2+1) + (Bx+C)(x+1)$$

$$1 = A(x^2+1) + Bx(x+1) + C(x+1) \ldots \text{(ii)}$$

Putting $x + 1 = 0$ i.e $x = -1$ in (ii), we get

$$1 = A[(-1)^2 + 1]$$

$$1 = A(1+1)$$

$$1 = 2A$$

$$\boxed{A = \frac{1}{2}}$$

Equating the coefficients of x^2 and x in equation (ii) we get

$$A + B = 0$$

$$\frac{1}{2} + B = 0$$

$$\Rightarrow \boxed{B = -\frac{1}{2}}$$

$$B + C = 0$$

$$-\frac{1}{2} + C = 0$$

$$\Rightarrow \boxed{C = \frac{1}{2}}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} - \frac{x-1}{2(1+x^2)}$$

Q.4 $\dfrac{9x-7}{(x+3)(x^2+1)}$

Solution: $\dfrac{9x-7}{(x+3)(x^2+1)}$

Let $\dfrac{9x-7}{(x+3)(x^2+1)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1}$ .....(i)

Multiplying both sides by (x+3)(x^2+1) we get

$$9x - 7 = A(x^2+1) + (Bx+C)(x+3)$$

$$9x - 7 = A(x^2+1) + Bx(x+3) + C(x+3) \ldots \text{(ii)}$$

Putting $x+3 = 0$ i.e $x = -3$ in (ii), we get

$$9(-3) - 7 = A[(-3)^2 + 1]$$

$$-27 - 7 = A(9+1)$$

$$-34 = 10A$$

$$\Rightarrow A = \frac{-34}{10} \Rightarrow \boxed{A = \frac{-17}{5}}$$

Equating coefficients of x^2 and x in equation (ii) we get

$$A + B = 0$$

$$\frac{-17}{5} + B = 0$$

$$\Rightarrow \boxed{B = \frac{17}{5}}$$

$$3B + C = 9$$

$$3\left(\frac{17}{5}\right) + C = 9$$

$$\frac{51}{5} + C = 9$$

$$C = 9 - \frac{51}{5}$$

$$C = \frac{45-51}{5}$$

$$\Rightarrow \boxed{C = \frac{-6}{5}}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{17x-6}{5(x^2+1)}$$

Q.5 $\dfrac{3x+7}{(x+3)(x^2+4)}$

Solution: $\dfrac{3x+7}{(x+3)(x^2+4)}$

Let $\dfrac{3x+7}{(x+3)(x^2+4)} = \dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+4}$ ... (i)

Multiplying both sides by $(x+3)(x^2+4)$ we get

$$3x + 7 = A(x^2+4) + (Bx+C)(x+3)$$

$$3x + 7 = A(x^2+4)+Bx(x+3)+C(x+3) \ldots \text{(ii)}$$

Putting $x + 3 = 0$ i.e $x = -3$ in (ii) we get

$$3(-3)+7 = A((-3)^2+4)$$
$$-9+7 = A(9+4)$$
$$-2 = 13A$$

$$\Rightarrow \boxed{A = \frac{-2}{13}}$$

Equating the coefficients of x^2 and x in equation (ii) we get

$$A + B = 0$$
$$\frac{-2}{13}+B = 0$$
$$B = \frac{2}{13}$$
$$\Rightarrow \boxed{B = \frac{2}{13}}$$

$$3B + C = 3$$
$$3\left(\frac{2}{13}\right)+C = 3$$
$$\frac{6}{13}+C = 3$$
$$C = 3-\frac{6}{13}$$
$$C = \frac{39-6}{13}$$
$$\Rightarrow \boxed{C = \frac{33}{13}}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{-2}{13(x+3)}+\frac{2x+33}{13(x^2+4)}$$

Q.6 $\dfrac{x^2}{(x+2)(x^2+4)}$

Solution: $\dfrac{x^2}{(x+2)(x^2+4)}$

Let $\dfrac{x^2}{(x+2)(x^2+4)} = \dfrac{A}{x+2}+\dfrac{Bx+C}{x^2+4}$ ....(i)

Multiplying both sides by $(x+2)(x^2+4)$ we get

$$x^2 = A(x^2+4) + (Bx+C)(x+2)$$

$$x^2 = A(x^2+4)+Bx(x+2)+C(x+2) \ldots \text{(ii)}$$

Putting $x + 2 = 0$ i.e $x = -2$ in (ii) we get

$$(-2)^2 = A[(-2)^2+4]$$
$$4 = A(4+4)$$
$$4 = 8A$$
$$\Rightarrow A = \frac{4}{8}$$
$$\boxed{A = \frac{1}{2}}$$

Equating the coefficients of x^2 and x in equation (ii) we get

$$A + B = 1$$
$$\frac{1}{2}+B = 1$$
$$B = 1-\frac{1}{2}$$
$$\Rightarrow \boxed{B = \frac{1}{2}}$$

$$2B + C = 0$$
$$\cancel{2}\left(\frac{1}{\cancel{2}}\right)+C = 0$$
$$1 + C = 0$$
$$\Rightarrow \boxed{C = -1}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)}+\frac{x-2}{2(x^2+4)}$$

Q.7 $\frac{1}{x^3+1}$

Solution: $\frac{1}{x^3+1}$

$$\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)}$$

Let $\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ .....(i)

Multiplying both sides by $(x+1)(x^2-x+1)$, we get

$1 = A(x^2-x+1) + (Bx+C)(x+1)$

$1 = A(x^2-x+1) + Bx(x+1) + C(x+1)$...(ii)

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$1 = A\,[(-1)^2-(-1)+1]$

$1 = A(1+1+1)$

$1 = 3A$

$\Rightarrow \boxed{A = \frac{1}{3}}$

Comparing the coefficients of x^2 and x in equation (ii) we get

$A + B = 0$

$\frac{1}{3} + B = 0$

$\Rightarrow B = \frac{-1}{3}$

$\boxed{B = \frac{-1}{3}}$

$-A + B + C = 0$

$-\frac{1}{3} - \frac{1}{3} + C = 0$

$\frac{-2}{3} + C = 0$

$\Rightarrow \boxed{C = \frac{2}{3}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{1}{3(x+1)} - \frac{x-2}{3(x^2-x+1)}$$

Q.8 $\frac{x^2+1}{x^3+1}$

Solution: $\frac{x^2+1}{x^3+1}$

$$\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)}$$

Let $\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ .....(i)

Multiplying both sides by $(x+1)(x^2-x+1)$, we get

$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1)$

$x^2+1 = A(x^2-x+1) + Bx(x+1) + C(x+1)$...(ii)

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$(-1)^2+1 = A\,[(-1)^2-(-1)+1]$

$1+1 = A(1+1+1)$

$2 = 3A$

$\Rightarrow \boxed{A = \frac{2}{3}}$

Equating the coefficients of x^2 and x in equation (ii) we get

$A + B = 1$

$\frac{2}{3} + B = 1$

$B = 1 - \frac{2}{3}$

$\Rightarrow \boxed{B = \frac{1}{3}}$

$-A + B + C = 0$

$\frac{-2}{3} + \frac{1}{3} + C = 0$

$\frac{-1}{3} + C = 0$

$\Rightarrow \boxed{C = \frac{1}{3}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{x^2+1}{x^3+1} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$$