EXERCISE 5.2

Q.1 If X = {1, 3, 5, 7, …., 19}
Y = {0, 2, 4, 6, 8, …, 20}
Z = {2, 3, 5, 7, 11, 13, 17, 19, 23}, then find the following:

(i) $X \cup (Y \cup Z)$ (ii) $(X \cup Y) \cup Z$

(iii) $X \cap (Y \cap Z)$ (iv) $(X \cap Y) \cap Z$

(v) $X \cup \cap Y \cap Z)$ (vi) $(X \cup Y) \cap (X \cup Z)$

(vii) $X \cap (Y \cup Z)$ (viii) $(X \cap Y) \cup (X \cap Z)$

Solution:

(i) $X \cup (Y \cup Z)$

$= X \cup (\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \cup \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23\}$

(ii) $(X \cup Y) \cup Z$

$= (\{1,3,5,7,\ldots,19\} \cup \{0,2,4,6,8,\ldots,20\}) \cup Z$

$= \{0, 1, 2, 3, 4, 5, 6, 7, \ldots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, \ldots, 20, 23\}$

(iii) $X \cap (Y \cap Z)$

$= X \cap (\{0,2,4,6, 8, \ldots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, \ldots, 19\} \cap \{2\}$

$= \phi$

(iv) $(X \cap Y) \cap Z$

$= (\{1,3,5,7, ..,19\} \cap \{0,2,4,6, 8,..,20\}) \cap Z$

$= \{\ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \phi$

(v) $X \cup (Y \cap Z)$

$= X \cup (\{0,2,4,6,8, \ldots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, \ldots, 19\} \cup \{2\}$

$= \{1, 2, 3 , 5, 7, \ldots, 19\}$

(vi) $(X \cup Y) \cap (X \cup Z)$

$X \cup Y = \{1,3,5, 7,\ldots, 19\} \cup \{0,2,4,6,8, \ldots, 20\}$

$= \{0, 1, 2, 3, 4, 5, \ldots, 20\}$

$X \cup Z = \{1,3,5,7, \ldots, 19\} \cup \{2,3,5,7,11,13,17, 19, 23\}$

$= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\}$

$(X \cup Y) \cap (X \cup Z)$

$= \{0,1,2,3,4, \ldots, 20\} \cap \{1,2,3,5,7,9,11,13,15,17,19,23\}$

$= \{1,2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

(vii) $X \cap (Y \cup Z)$

$X \cap (Y \cup Z)$

$= X \cap (\{0,2,4,6,8, \ldots,20\} \cup \{2,3,5,7,11,13, 17,19,23\})$

$= \{1,3,5,7, \ldots, 19\} \cap \{0,2,3,4,5,6,7,8, 10,11, 12,13,14,16,17,18,19,20,23\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

(viii) $(X \cap Y) \cup (X \cap Z)$

$X \cap Y = \{1,3,5,7, \ldots, 19\} \cap \{0,2,4,6,8, \ldots, 20\}$

$= \{\ \}$

$X \cap Z = \{1,3,5,7, \ldots, 19\} \cap \{2,3,5,7,11,13,17, 19,23\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

$(X \cap Y) \cup (X \cap Z) = \{\ \} \cup \{3,5,7,11,13, 17,19\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

Q. 2. If A = {1, 2, 3, 4, 5, 6}
B = {2, 4, 6, 8} C = {1, 4, 8} Prove the following identities:

(i) $A \cap B = B \cap A$

(ii) $A \cup B = B \cup A$

(iii) $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

(i) $A \cap B = B \cap A$

L.H.S = $A \cap B$

$= \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$

$= \{2, 4, 6\}$

R.H.S = $B \cap A$

$= \{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$

$= \{2, 4, 6\}$

L.H.S = R.H.S, so

$A \cap B = B \cap A$

(ii) $A \cup B = B \cup A$

L.H.S = $A \cup B$

$= \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

R.H.S = $B \cup A$

$= \{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

L.H.S = R.H.S,

So, $A \cup B = B \cup A$

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S = $A \cap (B \cup C)$

$= A \cap (\{2, 4, 6, 8\} \cup \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\}$

$= \{1, 2, 4, 6\}$

R.H.S = $(A \cap B) \cup (A \cap C)$

$A \cap B = \{1,2,3,4,5,6\} \cap \{2,4,6,8\}$

$= \{2, 4, 6\}$

$A \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\}$

$= \{1, 4\}$

$(A \cap B) \cup (A \cap C) = \{2, 4, 6\} \cup \{1, 4\}$

$= \{1, 2, 4, 6\}$

L.H.S = R.H.S

So, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S = $A \cap (B \cap C)$

$= A \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6\} \cup \{4, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

R.H.S = $(A \cup B) \cap (A \cup C)$

$A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

$A \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

$(A \cup B) \cap (A \cup C)$

$= \{1,2,3,4,5,6,8\} \cap \{1,2,3,4,5,6,8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

L.H.S = R.H.S,

So, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Q.3 If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} A = {1, 3, 5, 7, 9}, B = {2, 3, 5, 7} then verify the De Morgan's laws i.e., $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

Solution:

(i) $(A \cup B)' = A' \cap B'$

L.H.S = $(A \cup B)'$

$A \cup B = \{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\}$

$= \{1, 2, 3, 5, 7, 9\}$

$(A \cup B)' = U - (A \cup B)$

$= \{1,2,3,4, \ldots,10\} - \{1,2,3,5,7,9\}$

$= \{4, 6, 8, 10\}$.....(i)

R.H.S = $A' \cap B'$

$A' = U - A$

$= \{1, 2, 3, 4, \ldots, 10\} - \{1,3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

$B' = U - B$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$

$= \{4, 6, 8, 10\}$.....(ii)

From (i) and (ii)

L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

L.H.S = $(A \cap B)'$

$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$

$= \{3, 5, 7\}$

$(A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, \ldots, 10\} - \{3, 5, 7\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$.....(i)

R.H.S $= A' \cup B'$

$A' = U - A$

$= \{1,2,3,4,5,\ldots,10\} - \{1,3,5,7,9\}$

$= \{2, 4, 6, 8, 10\}$

$B' = U - B$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 3,5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{1, 4,6,8,9,10\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$.....(ii)

From (i) and (ii)

L.H.S = R.H.S

So $(A \cap B)' = A' \cup B'$

Q.4 If U = {1, 2, 3, ..., 20}

X = {1, 3, 7, 9, 15, 18, 20}

Y = {1, 3, 5, ..., 17} then show that,

(i) $X - Y = X \cap Y'$ (ii) $Y - X = Y \cap X'$

Solution:

(i) $X - Y = X \cap Y'$

L.H.S = X–Y

$= \{1, 3, 7, 9, 15, 18, 20\} -$

$\{1, 3, 5, 7, 9, 11, 13, 15, 17\}$

$= \{18, 20\}$.....(i)

R.H.S $= X \cap Y'$

$Y' = U - Y$

$= \{1,2,3,\ldots, 20\} - \{1,3,5,\ldots,17\}$

$= \{2,4,6,8,10,12,14,16,18,19,20\}$

$X \cap Y' = \{1, 3, 7, 9, 15, 18, 20\} \cap$

$\{2, 4, 6, 8, 10, 12, 14, 16, 18, 19, 20\}$

$= \{18, 20\}$.....(ii)

From (i) and (ii)

L.H.S = R.H.S,

So, $X - Y = X \cap Y'$

(ii) $Y - X = Y \cap X'$

L.H.S = Y–X

$= \{1,3,5,\ldots,17\} - \{1,3,7,9,15,18,20\}$

$= \{5, 11, 13, 17\}$.....(i)

R.H.S = $Y \cap X'$

$X' = U - X$

$= \{1,2,3, \ldots,20\} - \{1,3,7,9,15,18,20\}$

$= \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16,$

$17, 19\}$

$Y \cap X' = \{1, 3, 5, 7, 9, 11, 13, 15, 17\} \cap$

$\{2, 4, 5, 6, 8, 10, 11, 12, 13, 14,$

$16, 17, 19\}$

$= \{5, 11, 13, 17\}$.....(ii)

From (i) and (ii)

L.H.S = R.H.S

So, $Y - X = Y \cap X'$