

## EXERCISE 5.3

**Q.1** If  $U = \{1, 2, 3, 4, \dots, 10\}$   
 $A = \{1, 3, 5, 7, 9\}$ ,  $B = \{1, 4, 7, 10\}$   
 then verify the following questions:

**Solution:**

(i)  $A - B = A \cap B'$

$$\begin{aligned} \text{L.H.S} &= A - B = \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\} \\ &= \{3, 5, 9\} \dots\dots\dots (i) \end{aligned}$$

$$\text{R.H.S} = A \cap B'$$

$$\begin{aligned} B' &= U - B = \{1, 2, 3, \dots, 10\} - \{1, 4, 7, 10\} \\ &= \{2, 3, 5, 6, 8, 9\} \end{aligned}$$

$$\begin{aligned} A \cap B' &= \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\} \\ &= \{3, 5, 9\} \dots\dots\dots (ii) \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$A - B = A \cap B'$$

(ii)  $B - A = B \cap A'$

$$\begin{aligned} \text{L.H.S} &= B - A = \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\} \\ &= \{4, 10\} \dots\dots\dots (i) \end{aligned}$$

$$\text{R.H.S} = B \cap A'$$

$$\begin{aligned} A' &= U - A = \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\} \\ &= \{2, 4, 6, 8, 10\} \end{aligned}$$

$$\begin{aligned} B \cap A' &= \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\} \\ &= \{4, 10\} \dots\dots\dots (ii) \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$B - A = B \cap A'$$

(iii)  $(A \cup B)' = A' \cap B'$

$$\text{L.H.S} = (A \cup B)'$$

$$\begin{aligned} A \cup B &= \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\} \\ &= \{1, 3, 4, 5, 7, 9, 10\} \end{aligned}$$

$$\begin{aligned} (A \cup B)' &= U - (A \cup B) \\ &= \{1, 2, 3, \dots, 10\} - \{1, 3, 4, 5, 7, 9, 10\} \\ &= \{2, 6, 8\} \dots\dots\dots (i) \end{aligned}$$

$$\text{R.H.S} = A' \cap B'$$

$$\begin{aligned} A' &= U - A = \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\} \\ &= \{2, 4, 6, 8, 10\} \end{aligned}$$

$$\begin{aligned} B' &= U - B = \{1, 2, 3, \dots, 10\} - \{1, 4, 7, 10\} \\ &= \{2, 3, 5, 6, 8, 9\} \end{aligned}$$

$$\begin{aligned} A' \cap B' &= \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 6, 8, 9\} \\ &= \{2, 6, 8\} \dots\dots\dots (ii) \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

(iv)  $(A \cap B)' = A' \cup B'$

$$\text{L.H.S} = (A \cap B)'$$

$$\begin{aligned} A \cap B &= \{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\} \\ &= \{1, 7\} \end{aligned}$$

$$\begin{aligned} (A \cap B)' &= U - (A \cap B) \\ &= \{1, 2, 3, \dots, 10\} - \{1, 7\} \\ &= \{2, 3, 4, 5, 6, 8, 9, 10\} \dots\dots (i) \end{aligned}$$

$$\text{R.H.S} = A' \cup B'$$

$$\begin{aligned} A' &= U - A = \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\} \\ &= \{2, 4, 6, 8, 10\} \end{aligned}$$

$$\begin{aligned} B' &= U - B = \{1, 2, 3, \dots, 10\} - \{1, 4, 7, 10\} \\ &= \{2, 3, 5, 6, 8, 9\} \end{aligned}$$

$$\begin{aligned} A' \cup B' &= \{2, 4, 6, 8, 10\} \cup \{2, 3, 5, 6, 8, 9\} \\ &= \{2, 3, 4, 5, 6, 8, 9, 10\} \dots\dots (ii) \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

(v)  $(A - B)' = A' \cup B$

$$\text{L.H.S} = (A - B)'$$

$$\begin{aligned} A - B &= \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\} \\ &= \{3, 5, 9\} \end{aligned}$$

$$\begin{aligned} (A - B)' &= U - (A - B) \\ &= \{1, 2, 3, \dots, 10\} - \{3, 5, 9\} \\ &= \{1, 2, 4, 6, 7, 8, 10\} \dots\dots (i) \end{aligned}$$

$$\text{R.H.S} = A' \cup B$$

$$\begin{aligned} A' &= U - A = \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\} \\ &= \{2, 4, 6, 8, 10\} \end{aligned}$$

$$\begin{aligned} A' \cup B &= \{2, 4, 6, 8, 10\} \cup \{1, 4, 7, 10\} \\ &= \{1, 2, 4, 6, 7, 8, 10\} \dots\dots (ii) \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

(vi)  $(B - A)' = B' \cup A$

$$\text{L.H.S} = (B - A)'$$

$$\begin{aligned} B - A &= \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\} \\ &= \{4, 10\} \end{aligned}$$

$$\begin{aligned} (B - A)' &= U - (B - A) \\ &= \{1, 2, 3, \dots, 10\} - \{4, 10\} \\ &= \{1, 2, 3, 5, 6, 7, 8, 9\} \dots\dots (i) \end{aligned}$$

$$\text{R.H.S} = B' \cup A$$

$$\begin{aligned} B' &= U - B = \{1, 2, 3, 4, \dots, 10\} - \{1, 4, 7, 10\} \\ &= \{2, 3, 5, 6, 8, 9\} \end{aligned}$$

$$\begin{aligned} B' \cup A &= \{2, 3, 5, 6, 8, 9\} \cup \{1, 3, 5, 7, 9\} \\ &= \{1, 2, 3, 5, 6, 7, 8, 9\} \dots\dots (ii) \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

Q.2 If  $A = \{1, 3, 5, 7, 9\}$   $B = \{1, 4, 7, 10\}$   
 $C = \{1, 5, 8, 10\}$

Then verify the following:

Solution:

(i)  $(A \cup B) \cup C = A \cup (B \cup C)$

$$\begin{aligned} \text{L.H.S} &= (A \cup B) \cup C \\ &= (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}) \cup \{1, 5, 8, 10\} \\ &= \{1, 3, 4, 5, 7, 9, 10\} \cup \{1, 5, 8, 10\} \\ &= \{1, 3, 4, 5, 7, 8, 9, 10\} \quad \dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= A \cup (B \cup C) \\ &= \{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\} \cup \{1, 5, 8, 10\}) \\ &= \{1, 3, 5, 7, 9\} \cup \{1, 4, 5, 7, 8, 10\} \\ &= \{1, 3, 4, 5, 7, 8, 9, 10\} \quad \dots\dots (ii) \end{aligned}$$

From (i) and (ii)

L.H.S = R.H.S

(ii)  $(A \cap B) \cap C = A \cap (B \cap C)$

$$\begin{aligned} \text{L.H.S} &= (A \cap B) \cap C \\ &= (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\} \\ &= \{1, 7\} \cap \{1, 5, 8, 10\} \\ &= \{1\} \quad \dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= A \cap (B \cap C) \\ &= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\}) \\ &= \{1, 3, 5, 7, 9\} \cap \{1, 10\} \\ &= \{1\} \quad \dots\dots (ii) \end{aligned}$$

From (i) and (ii)

L.H.S = R.H.S

(iii)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$\begin{aligned} \text{L.H.S} &= A \cup (B \cap C) \\ &= \{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\}) \\ &= \{1, 3, 5, 7, 9\} \cup \{1, 10\} \\ &= \{1, 3, 5, 7, 9, 10\} \quad \dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= (A \cup B) \cap (A \cup C) \\ A \cup B &= \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\} \\ &= \{1, 3, 4, 5, 7, 9, 10\} \end{aligned}$$

$$\begin{aligned} A \cup C &= \{1, 3, 5, 7, 9\} \cup \{1, 5, 8, 10\} \\ &= \{1, 3, 5, 7, 8, 9, 10\} \end{aligned}$$

$$\begin{aligned} \text{Now } (A \cup B) \cap (A \cup C) &= \{1, 3, 4, 5, 7, 9, 10\} \cap \{1, 3, 5, 7, 8, 9, 10\} \\ &= \{1, 3, 5, 7, 9, 10\} \quad \dots\dots (ii) \end{aligned}$$

From (i) and (ii)

L.H.S = R.H.S

(iv)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$\begin{aligned} \text{L.H.S} &= A \cap (B \cup C) \\ &= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cup \{1, 5, 8, 10\}) \\ &= \{1, 3, 5, 7, 9\} \cap \{1, 4, 5, 7, 8, 10\} \\ &= \{1, 5, 7\} \quad \dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= (A \cap B) \cup (A \cap C) \\ A \cap B &= \{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\} \\ &= \{1, 7\} \\ A \cap C &= \{1, 3, 5, 7, 9\} \cap \{1, 5, 8, 10\} \\ &= \{1, 5\} \end{aligned}$$

$$\begin{aligned} \text{Now } (A \cap B) \cup (A \cap C) &= \{1, 7\} \cup \{1, 5\} \\ &= \{1, 5, 7\} \dots\dots (ii) \end{aligned}$$

From (i) and (ii)

L.H.S = R.H.S

Q.3 If  $U = N$ , then verify De-Morgan's laws by using:

$A = \phi, B = P$

Solution:

$$\begin{aligned} A &= \{ \} \\ B &= \{2, 3, 5, 7, \dots\} \\ U &= \{1, 2, 3, 4, 5, 6, 7, \dots\} \end{aligned}$$

(i)  $(A \cup B)' = A' \cap B'$

$$\begin{aligned} \text{L.H.S} &= (A \cup B)' \\ A \cup B &= \{ \} \cup \{2, 3, 5, 7, \dots\} \\ &= \{2, 3, 5, 7, \dots\} \\ (A \cup B)' &= U - (A \cup B) \\ &= \{1, 2, 3, 4, 5, 6, 7, \dots\} - \{2, 3, 5, 7, \dots\} \\ &= \{1, 4, 6, \dots\} \quad \dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= A' \cap B' \\ A' &= U - A = \{1, 2, 3, 4, 5, 6, 7, \dots\} - \{ \} \\ &= \{1, 2, 3, 4, 5, 6, 7, \dots\} \\ B' &= U - B = \{1, 2, 3, 4, 5, 6, 7, \dots\} - \{2, 3, 5, 7, \dots\} \\ &= \{1, 4, 6, \dots\} \\ A' \cap B' &= \{1, 2, 3, 4, 5, 6, 7, \dots\} \cap \{1, 4, 6, \dots\} \\ &= \{1, 4, 6, \dots\} \quad \dots\dots (ii) \end{aligned}$$

From (i) and (ii)

L.H.S = R.H.S

(ii)  $(A \cap B)' = A' \cup B'$

L.H.S =  $(A \cap B)'$

$(A \cap B) = \{ \} \cap \{2, 3, 5, 7, \dots\}$

=  $\{ \}$

$(A \cap B)' = U - (A \cap B)$

=  $\{1, 2, 3, 4, 5, 6, 7, \dots\} - \{ \}$

=  $\{1, 2, 3, 4, 5, 6, 7, \dots\}$ ..... (i)

R.H.S =  $A' \cup B'$

$A' = U - A = \{1, 2, 3, 4, \dots\} - \{ \}$

=  $\{1, 2, 3, 4, \dots\}$

$B' = U - B = \{1, 2, 3, 4, 5, 6, 7, \dots\} - \{2, 3, 5, 7, \dots\}$

=  $\{1, 4, 6, \dots\}$

$A' \cup B' = \{1, 2, 3, 4, 5, 6, 7, \dots\} \cup \{1, 4, 6, \dots\}$

=  $\{1, 2, 3, 4, \dots\}$ ..... (ii)

From (i) and (ii)

L.H.S = R.H.S

**Q.4** If  $U = \{1, 2, 3, 4, \dots, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$  and  $B = \{2, 3, 4, 5, 8\}$  then prove the following questions by Venn Diagram.

**Solution:**  $A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 4, 5, 8\} = \{3, 5\}$

So given sets A and B are overlapping sets

(i)  $A - B = A \cap B'$

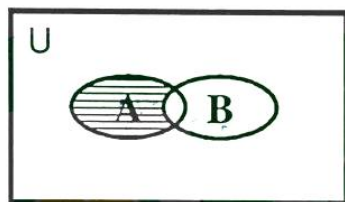


Fig: 1 (A-B)

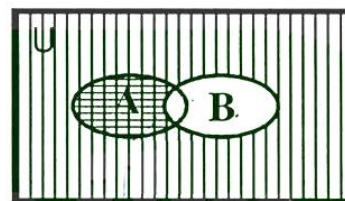


Fig: 2  $A \cap B'$

- A - B is shown by horizontal line segments in fig. 1.
- B' is shown by vertical line segments and squares in fig. 2.
- $A \cap B'$  is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus  $A - B = A \cap B'$

(ii)  $B - A = B \cap A'$

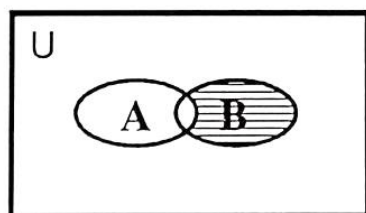


Fig. 1 (B - A)

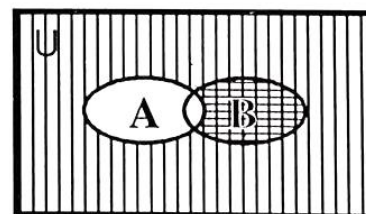


Fig: 2  $B \cap A'$

- B - A is shown by horizontal line segments in fig. 1.
- A' is shown by vertical line segments and squares in fig. 2.
- $B \cap A'$  is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus  $B - A = B \cap A'$ .

(iii)  $(A \cup B)' = A' \cap B'$  (De-Morgan's Law)

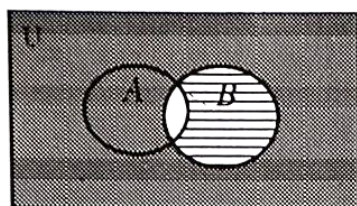


Fig. 1

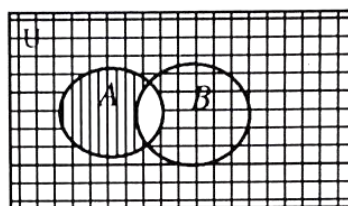


Fig. 2

- $A \cup B$  is shown by horizontal line segments in Fig. 1.
- $(A \cup B)'$  is shown by shaded area in Fig. 1.
- $A'$  is shown by horizontal line segments and squares in Fig. 2.
- $B'$  is shown by vertical line segments and squares in Fig. 2.
- $A' \cap B'$  is shown by squares in Fig. 2.

Shaded area shown in Fig. 1 and square area shown in Fig. 2 are equal.

thus  $(A \cup B)' = A' \cap B'$

(iv)  $(A \cap B)' = A' \cup B'$

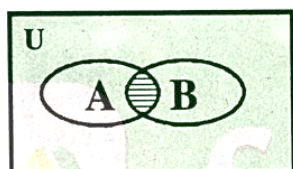


Fig. 1

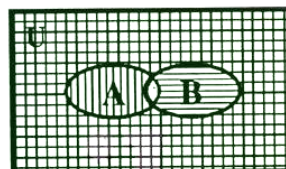


Fig. 2

- $A \cap B$  is shown by horizontal line segments in Fig. 1.
- $(A \cap B)'$  is shown by shaded area in Fig. 1.
- $A'$  is shown by horizontal line segments and squares in Fig. 2.
- $B'$  is shown by vertical line segments and squares in Fig. 2.
- $A' \cup B'$  is shown by squares, horizontal and vertical line segments in Fig. 2.

Shaded area shown in Fig. 1 and area of squares, vertical and horizontal line segments shown in Fig. 2 are equal. thus  $(A \cap B)' = A' \cup B'$

(v)  $(A - B)' = A' \cup B$

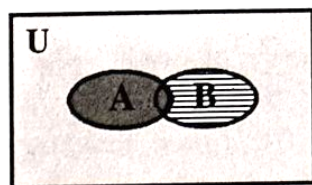


Fig. 1

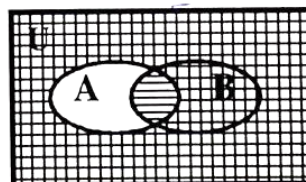


Fig. 2

- $A - B$  is shown by horizontal line segments in Fig. 1.
- $(A - B)'$  is shown by shaded area in Fig. 1.
- $A'$  is shown by squares Fig. 2.
- $A' \cup B$  is shown by squares and horizontal line segments in Fig. 2.

Shaded area in Fig. 1 and area of squares & horizontal line segments in Fig. 2 are equal.

thus  $(A - B)' = A' \cup B$

(vi)  $(B-A)' = B' \cup A$

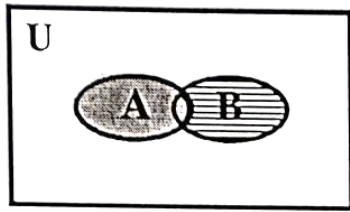


Fig. 1

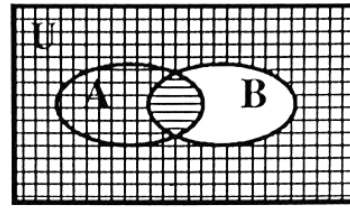


Fig. 2

- $B-A$  is shown by horizontal line segments in Fig. 1.
  - $(B-A)'$  is shown by shaded area in Fig.1.
  - $B'$  is shown by squares in Fig. 2.
  - $B' \cup A$  is shown by squares and horizontal line segments in Fig. 2.
- Shaded area in Fig. 1 and area of squares & vertical line segments in Fig. 2 are equal.  
 thus  $(B-A)' = B' \cup A$