EXERCISE 5.5

Q.1 If L = {a, b, c}, M = {3, 4}, then find two binary relations of $L \times M$ and $M \times L$.

Solution: $L = \{a, b, c\}$, $M = \{3, 4\}$

$L \times M = \{a, b, c\} \times \{3, 4\}$

$= \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$

Two binary Relations:

$R_1 = \{(a, 3), (a, 4)\}$

$R_2 = \{(b, 4), (c, 3), (c, 4)\}$

$M \times L = \{3, 4\} \times \{a, b, c\}$

$= \{(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)\}$

Two binary Relations:

$R_1 = \{(3, a), (3, b)\}$

$R_2 = \{(4, b), (4, c)\}$

Q.2 If Y = {-2, 1, 2}, then make two binary relations for $Y \times Y$. Also find their domain and range.

Solution: $Y = \{-2, 1, 2\}$

$Y \times Y = \{-2, 1, 2\} \times \{-2, 1, 2\}$

$= \{(-2,-2), (-2,1), (-2,2), (1,-2), (1,1), (1,2),$
$(2,-2,)\ (2,1), (2,2)\}$

$R_1 = \{(-2, 1), (-2, 2), (1, -2)\}$

Domain $R_1 = \{-2, 1\}$

Range $R_1 = \{1, 2, -2\}$

$R_2 = \{(1, 1), (2, -2), (2, 2)\}$

Domain $R_2 = \{1, 2\}$

Range $R_2 = \{1, -2, 2\}$

Q.3 If L = {a, b, c} and M = {d, e, f, g}, then find two binary relations in each.

(i) $L \times L$ (ii) $L \times M$ (iii) $M \times M$

Solution: $L = \{a, b, c\}$, $M = \{d, e, f, g\}$

(i) $L \times L = \{a, b, c\} \times \{a, b, c\}$

$L \times L = \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}$

Two binary Relations:

$R_1 = \{(a, a), (a, b), (a, c)\}$

$R_2 = \{(b, c), (c, a), (c, b)\}$

(ii) L×M

$L \times M = \{a, b, c\} \times \{d, e, f, g\}$

$= \{(a,d),(a,e),(a,f),(a,g),(b,d),(b,e),(b,f),(b,g),(c,d),(c,e),(c,f),(c,g)\}$

Two binary Relations:

$R_1 = \{(a, d), (a, e), (a, f)\}$

$R_2 = \{(b, d), (b, e), (b, f)\}$

(iii) M×M

$M \times M = \{d, e, f, g\} \times \{d, e, f, g\}$

$= \{(d,d),(d,e),(d,f),(d,g),(e,d),(e,e),(e,f),(e,g),(f,d),(f,e),(f,f),(f,g),(g,d),(g,e),(g,f),(g,g)\}$

Two binary Relations:

$R_1 = \{(d, e), (d, f), (d, g)\}$

$R_2 = \{(f, d), (g, d)\}$

Q.4 If set M has 5 elements then find the number of binary relations in M.

No. of binary relations in $M = 2^{5\times5} = 2^{25}$

Q.5 If $L = \{x | x \in N \wedge x \leq 5\}$, $M = \{y | y \in P \wedge y < 10\}$, then make the following relations from L to M. Also write Domain and Range of each Relation.

(i) $R_1 = \{(x, y) \mid y < x\}$,

(ii) $R_2 = \{(x, y) \mid y = x\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

Solution

$L = \{1, 2, 3, 4, 5\}$,

$M = \{2, 3, 5, 7\}$

$L \times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),(4,7),(5,2),(5,3),(5,5),(5,7)\}$

(i) $R_1 = \{(x, y) \mid y < x\}$

$R_1 = \{(3, 2), (4, 2), (4, 3), (5, 2), (5, 3)\}$

Domain $R_1 = \{3, 4, 5\}$

Range $R_1 = \{2, 3\}$

(ii) $R_2 = (x, y) \mid y = x\}$

$R_2 = \{(2, 2), (3, 3), (5, 5)\}$

Domain $R_2 = \{2, 3, 5\}$

Range $R_2 = \{2, 3, 5\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

$R_3 = \{(1, 5), (3, 3), (4, 2)\}$

Domain $R_3 = \{1, 3, 4\}$

Range $R_3 = \{5, 3, 2\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

$R_4 = \{(1, 3), (3, 5), (5, 7)\}$

Domain $R_4 = \{1, 3, 5\}$

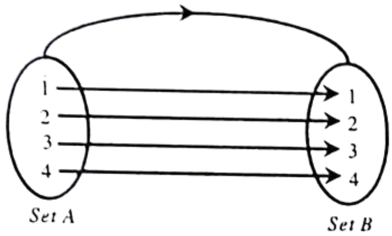
Range $R_4 = \{3, 5, 7\}$

Q.6 Indicate relations, into function, one-one function, onto function, and bijective function from the following. Also find their domain and the range.

$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Domain $R_1 = \{1, 2, 3, 4\}$

Range $R_1 = \{1, 2, 3, 4\}$

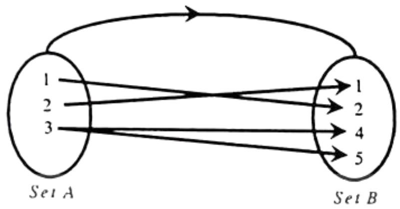


It is a bijective function.

(ii) $R_2 = \{(1, 2), (2, 1), (3, 4), (3, 5)\}$

Domain $R_2 = \{1, 2, 3\}$

Range $R_2 = \{1, 2, 4, 5\}$

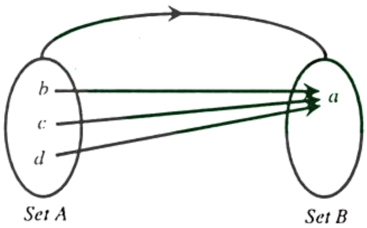


It is a relation. As 3 has no distinct image.

(iii) $R_3 = \{(b, a), (c, a), (d, a)\}$

Domain $R_3 = \{b, c, d\}$

Range $R_3 = \{a\}$

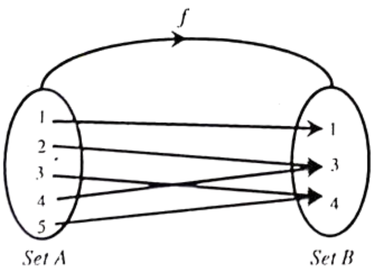


It is an onto function.

(iv) $R_4 = \{(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)\}$

Domain $R_4 = \{1, 2, 3, 4, 5\}$

Range $R_4 = \{1, 3, 4\}$



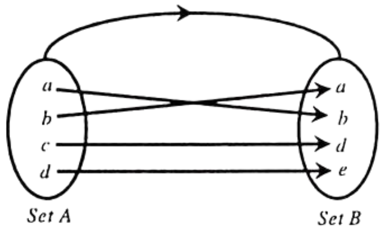
It is an onto function.

(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$

Domain $R_5 = \{a, b, c, d\}$

Range $R_5 = \{a, b, d, e\}$

It is a bijective function.

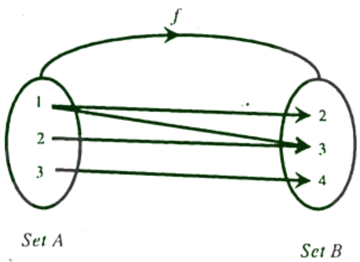


(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$

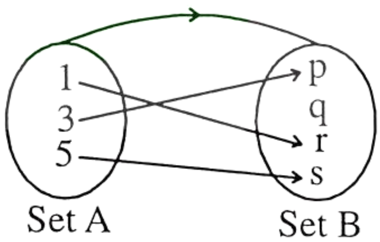
Domain $R_6 = \{1, 2, 3\}$

Range $R_6 = \{2, 3, 4\}$

It is a relation. As 1 has no distinct image.



(vii) $R_7 =$

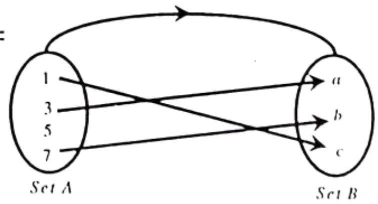


Domain $R_7 = \{1, 3, 5\}$

Range $R_7 = \{p, r, s\}$

R_7 is one-one function.

(viii) $R_8 =$



Domain $R_8 = \{1, 3, 7\}$

Range $R_8 = \{a, b, c\}$

It is a relation. As 5 has no distinct image.