EXERCISE 6.3

Q.1 What do you understand by Dispersion?

Dispersion: Statistically, dispersion means the spread or scatterness of observations in a data set.

Q.2 How do you define measures of dispersion?

Measures of Dispersion:

The measures or techniques that are used to determine the degree or extent of variation in a data set are called Measure of Dispersion.

For example Range, Variance and Standard Deviation etc.

Q.3 Define range, standard deviation and variance?

Definitions:

Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula.

Range =
$$X_{max} - X_{min} = X_m - X_o$$
 Where,

$$X_{max} = X_m$$
: the maximum, observation.

$$X_{min} = X_o$$
: the minimum observation.

Variance:

Variance is defined as the mean of the squared deviation of X_i , (i=1,2,3,..., n) observations from their arithmetic mean.

Variance from Ungrouped Data

• Definitional Formula

Variance of x = var (X) =
$$S^2 = \frac{\sum (x - \overline{x})^2}{n}$$

• Computational formula:

Var (x) =
$$S^2 = \frac{\sum x^2}{n} - (\frac{\sum x}{n})^2$$

Variance from Grouped Data

• Definitional Formula

Var (x) =
$$S^2 = \frac{\sum f(x - \bar{x})^2}{\sum f}$$

Computational Formula

$$Var(x) = S^{2} = \frac{\sum fx^{2}}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^{2}$$

Standard Deviation:

Standard deviation is defined as the positive square root of mean of the squared deviations of X_i , (i= 1,2,3...., n) observations from their arithmetic mean.

Standard Deviation from Un-Grouped Data

• Definitional Formula

S. D(X) =
$$S = \sqrt{\frac{\sum (x - \overline{x})^2}{n}}$$

• Computational Formula

S. D (x) = S=
$$\sqrt{\frac{\sum x^2}{n} - (\frac{\sum x}{n})^2}$$

Standard Deviation from Grouped Data

Definitional Formula

S.D (x) = S =
$$\sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

• Computational Formula

S.D (x) = S =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Q.4 The salaries of five teachers in Rupees are as follows. 11500, 12400, 15000, 14500, 14800

Find range and standard deviation

Solution

Maximum value = X_{max} = Rs. 15000

Minimum value = X_{min} = Rs. 11500

Range (R)=?

Range $=X_{max} - X_{min}$

R = 15000 - 11500

Range = Rs. 3500

Standard Deviation:

Let X = 11500, 12400, 15000, 14500, 14800

We know that
$$\frac{1}{X} = \frac{\sum X}{n} = \frac{68200}{5} = 13640$$

X	$x-\overline{x}$	$(x-x)^2$	
11500	11500 - 13640 = -2140	4579600	
12400	12400 - 13640 = -1240	1537600	
15000	15000 - 13640 = 1360	1849600	
14500	14500 - 13640 = 860	739600	
14800	14800 - 13640 = 1160	1345600	
$\sum_{X} = 68200$	$\sum_{x=0}^{\infty} (x-x)^2 = 10052000$		

Standard Deviation:

$$S = \sqrt{\frac{\sum (x - \overline{x})^2}{n}}$$

$$S = \sqrt{\frac{10052000}{5}}$$

$$S = \sqrt{2010400}$$

$$S = 1417.88575$$

Q. 5

a. Find the standard deviation "S" of each set of numbers.

- (i) 12, 6, 7, 3, 15, 10, 18, 5
- (ii) 9, 3, 8, 8, 9, 8, 9, 18

b. Calculate variance for the data:

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

(i) Solution:

$$X = 12, 6, 7, 3, 15, 10, 18, 5$$

Number of observations = n = 8

X	X^2
3	9
5	25
6	36
7	49
10	100
12	144
15	225
18	324
$\sum x = 76$	$\sum x^2 = 912$

$$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$S = \sqrt{\frac{912}{8} - \left(\frac{76}{8}\right)^2}$$

$$S = \sqrt{114 - 90.25}$$

$$S = \sqrt{23.75}$$

$$S = 4.87$$

(ii) Solution:

X = 9, 3, 8, 8, 9, 8, 9, 18

Number of observations = n = 8

X	X^2				
3	9				
8	64				
8	64				
8	64 64				
9	81				
9	81				
9	81				
18	324				
$\sum x = 72$	$\sum x^2 = 768$				

We know that.

$$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$S = \sqrt{\frac{768}{8} - \left(\frac{72}{8}\right)^2}$$

$$S = \sqrt{96 - 81}$$

$$S = \sqrt{15}$$

$$\Rightarrow$$
 S= 3.87

(b). Calculate variance for the data

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

Solution:

Number of Observations = n = 10

X	X^2
10	100
8	64
9	81
7	49
5	25
12	144
8	64
6	36
8	64
2	4
$\sum x = 75$	$\sum x^2 = 631$

We know that,

$$S^{2} = \frac{\sum x^{2}}{n} - \left(\frac{\sum x}{n}\right)^{2}$$

$$S^{2} = \frac{631}{10} - \left(\frac{75}{10}\right)^{2}$$

$$S^{2} = 63.1 - (7.5)^{2}$$

$$S^{2} = 63.1 - 56.25 \implies S^{2} = 6.85$$

Q. 6. The lengths of 32 items are given below. Find the mean length and standard deviation of the distribution.

Length	20–22	23–25	26–28	29-31	32-34
f	3	6	12	9	2

Solution:

Solution	•				
Length	(f)	Class Marks (x)	fx	x ²	fx^2
20-22	3	21	63	441	1323
23-25	6	24	144	576	3456
26-28	12	27	324	729	8748
29-31	9	. 30	270	900	8100
32-34	2	33	66	1089	2178
$\sum f$	=32	$\sum fx$	=867	$\sum fx^2$	= 23805

(i) Arithmetic Mean

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{867}{32} = 27.093$$

(ii) Standard Deviation

$$S = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$S = \sqrt{\frac{23805}{32} - \left(\frac{867}{32}\right)^2}$$

$$S = \sqrt{743.90 - (27.093)^2}$$

$$S = \sqrt{743.90 - 734.03065}$$

$$S = \sqrt{9.86935}$$

$$S = 3.1415$$

Q. 7 For the following distribution of marks calculate range.

Marks in percentage	Frequency / No of students
31 – 40	28
41 – 50	31
51 – 60	12
61 – 70	9
71 – 75	5 ·

Solution:

Marks in percentage	Class Boundaries	Frequency / No of students
31 – 40	30.5 – 40.5	28
41 – 50	40.5 – 50.5	31
51 - 60	50.5 – 60.5	12
61 – 70	60.5 – 70.5	9
71 – 75	70.5 – 75.5	5

Upper class boundary of last class = 75.5Lower class boundary of 1^{st} class = 30.5

Range = ?

Range (R) = 75.5 - 30.5R = 45