EXERCISE 7.4

In problem 1— 6, simply each expression to single trigonometric function:

Q.1. $\dfrac{\sin^2 x}{\cos^2 x}$

Solution: $\dfrac{\sin^2 x}{\cos^2 x} = \tan x^2$

Q.2. $\tan x \sin x \sec x$

Solution: $\tan x \sin x \sec x$

$= \dfrac{\sin x}{\cos x} . \sin x . \dfrac{1}{\cos x}$

$= \dfrac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

Q.3. $\dfrac{\tan x}{\sec x}$

Solution: $\dfrac{\tan x}{\sec x} = \tan x \div \sec x$

$= \dfrac{\sin x}{\cos x} \div \dfrac{1}{\cos x}$

$= \dfrac{\sin x}{\cancel{\cos x}} \times \cancel{\cos x}$

$= \sin x$

Q.4. $1 - \cos^2 x$

Solution: $1 - \cos^2 x$

$= \sin^2 x + \cancel{\cos^2} x - \cancel{\cos^2} x$

$= \sin^2 x$

Q.5. $\sec^2 x - 1$

Solution: $\sec^2 x - 1$

$= 1 + \tan^2 x - 1$

$= \tan^2 x$

Q.6. $\sin^2 x . \cot^2 x$

Solution:

$\sin^2 x . \cot^2 x$

$= \cancel{\sin^2} x \dfrac{\cos^2 x}{\cancel{\sin^2} x}$

$= \cos^2 x$

In problem 7 — 24, verify the identities

Q.7. $(1-\sin\theta)(1+\sin\theta) = \cos^2\theta$

Solution:

L.H.S $= (1 - \sin\theta)(1 + \sin\theta)$

$= (1)^2 - (\sin\theta)^2$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

L.H.S = R.H.S

Q.8. $\dfrac{\sin\theta + \cos\theta}{\cos\theta} = 1 + \tan\theta$

Solution: Let

L.H.S $= \dfrac{\sin\theta + \cos\theta}{\cos\theta}$

$= \dfrac{\cos\theta + \sin\theta}{\cos\theta}$

$= \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= 1 + \tan\theta$ $\quad \because \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

L.H.S = R.H.S

Q.9. **$(\tan\theta + \cot\theta)\tan\theta = \sec^2\theta$**

Solution: Let

L.H.S $= (\tan\theta + \cot\theta)\tan\theta$

$= \tan^2\theta + \cot\theta . \tan\theta$

$= \tan^2\theta + \frac{1}{\cancel{\tan\theta}} . \cancel{\tan\theta}$

$= 1 + \tan^2\theta$

$= \sec^2\theta \qquad (1 + \tan^2\theta = \sec^2\theta)$

L.H.S = R.H.S

Q.10. **$(\cot\theta + \text{cosec}\,\theta)(\tan\theta - \sin\theta) = \sec\theta - \cos\theta$**

Solution: Let

L.H.S $= (\cot\theta + \text{cosec}\,\theta)(\tan\theta - \sin\theta)$

$= \frac{1}{\tan\theta} + \frac{1}{\sin\theta} \quad (\tan\theta - \sin\theta)$

$= \frac{\sin\theta + \tan\theta}{\tan\theta . \sin\theta} \quad (\tan\theta - \sin\theta)$

$= \frac{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}{\tan\theta . \sin\theta}$

$= \frac{(\tan\theta)^2 - (\sin\theta)^2}{\tan\theta . \sin\theta}$

$= \frac{\tan^2\theta - \sin^2\theta}{\tan\theta . \sin\theta}$

$= \frac{\tan^{\cancel{2}}\theta}{\cancel{\tan\theta} . \sin\theta} - \frac{\sin^{\cancel{2}}\theta}{\tan\theta . \cancel{\sin\theta}}$

$= \frac{\tan\theta}{\sin\theta} - \frac{\sin\theta}{\tan\theta}$

$= (\tan\theta \div \sin\theta) - (\sin\theta \div \tan\theta)$

$= \frac{\sin\theta}{\cos\theta} \div \sin\theta - \sin\theta \div \frac{\sin\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta} - \sin\theta \times \frac{\cos\theta}{\sin\theta}$

$= \sec\theta - \cos\theta$

L.H.S = R.H.S

Q.11. **$\frac{\sin\theta + \cos\theta}{\tan^2\theta - 1} = \frac{\cos^2\theta}{\sin\theta - \cos\theta}$**

Solution: Let

L.H.S $= \frac{\sin\theta + \cos\theta}{\tan^2\theta - 1}$

$= (\sin\theta + \cos\theta) \div (\tan^2\theta - 1)$

$= (\sin\theta + \cos\theta) \div \frac{\sin^2\theta}{\cos^2\theta} - 1$

$= (\sin\theta + \cos\theta) \div \frac{\sin^2 - \cos^2}{\cos^2}$

$= (\sin\theta + \cos\theta) \times \frac{\cos^2}{(\sin^2\theta - \cos^2\theta)}$

$= \frac{\cancel{(\sin\theta + \cos\theta)} \times \cos^2\theta}{\cancel{(\sin\theta + \cos\theta)}(\sin\theta - \cos\theta)}$

$= \frac{\cos^2\theta}{\sin\theta - \cos\theta}$

L.H.S = R.H.S

Q.12. $\frac{\cos^2\theta}{\sin\theta}+\sin\theta = \text{cosec}\,\theta$

Solution: Let

L.H.S $= \frac{\cos^2\theta}{\sin\theta}+\sin\theta$

$= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$= \text{cosec}\,\theta$

L.H.S = R.H.S

Q.13. $\sec\theta - \cos\theta = \tan\theta\,.\sin\theta$

Solution: Let

L.H.S $= \sec\theta - \cos\theta$

$= \frac{1}{\cos\theta} - \cos\theta$

$= \frac{1-\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta}\sin\theta$

$= \tan\theta\,.\,\sin\theta$

L.H.S = R.H.S

Q.14. $\frac{\sin^2\theta}{\cos\theta} + \text{eos}\theta = \sec\theta$

Solution: Let

L.H.S $= \frac{\sin^2\theta}{\cos\theta}+\cos\theta$

$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

L.H.S = R.H.S

Q.15. $\tan\theta + \cot\theta = \sec\theta.\,\text{cosec}\theta$

Solution: Let

L.H.S $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta.\sin\theta}$

$= \frac{1}{\cos\theta.\sin\theta} \quad (\sin^2\theta+\cos^2\theta=1)$

$= \frac{1}{\cos\theta}.\frac{1}{\sin\theta}$

$= \sec\theta.\text{cosec}\theta$

L.H.S = R.H.S

Q.16. $(\tan\theta+\cot\theta)(\cos\theta+\sin\theta) = \sec\theta+\text{cosec}\theta$

Solution: Let

L.H.S $= (\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$

$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}(\cos\theta+\sin\theta)$

$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta.\sin\theta}(\cos\theta+\sin\theta)$

$= \frac{1}{\cos\theta.\sin\theta}(\cos\theta+\sin\theta)$

$= \frac{\cos\theta+\sin\theta}{\cos\theta.\sin\theta}$

$= \frac{\cos\theta}{\cos\theta.\sin\theta}+\frac{\sin\theta}{\cos\theta.\sin\theta}$

$= \frac{1}{\sin\theta}+\frac{1}{\cos\theta}$

$= \text{cosec}\theta + \sec\theta$

$= \sec\theta + \text{cosec}\theta$

L.H.S = R.H.S

Q.17. $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$

Solution: Let

L.H.S $= \sin\theta(\tan\theta + \cot\theta)$

$= \sin\theta \; \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$= \sin\theta \; \frac{\sin^2\theta+\cos^2\theta}{\cos\theta.\sin\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

L.H.S = R.H.S

Q.18. $\frac{1+\cos}{\sin}+\frac{\sin}{1+\cos}=2\text{cosec}$

Solution: Let

$$L.H.S = \frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)^2+(\sin\theta)^2}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{(1)^2+2(1)(\cos\theta)+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1\ \ 2\cos\theta+1}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2}{\sin\theta}$$

$= 2\text{cosec}\theta$

L.H.S = R.H.S

Q.19. $\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta} = 2\text{cosec}^2\theta$

Solution:

$$L.H.S = \frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}$$

$$= \frac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{2}{(1)^2-(\cos^2\theta)}$$

$$= \frac{2}{1-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

$= 2\ \text{cosec}^2\theta$

L.H.S = R.H.S

Q.20. $\frac{1+\sin}{1-\sin}-\frac{1-\sin}{1+\sin}=4\tan\ \sec$

Solution: Let

$$L.H.S = \frac{1+\sin\theta}{1-\sin\theta}=\frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2-(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{(1+\sin^2\theta+2\sin\theta)-(1+\sin^2\theta-2\sin\theta)}{(1)^2-(\sin\theta)^2}$$

$$= \frac{1+\sin^2\theta+2\sin\theta-1-\sin^2\theta+2\sin\theta}{1-\sin^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta}$$

$$= \frac{4\sin\theta}{\cos\theta.\cos\theta}$$

$$= 4\frac{\sin\theta}{\cos\theta}.\frac{1}{\cos\theta}$$

$= 4\tan\theta.\ \sec\theta$

L.H.S = R.H.S

Q.21. $\sin^3\theta = \sin\theta - \sin\theta.\cos^2\theta$

Solution: Let

L.H.S $= \sin^3\theta$

$= \sin\theta\ .\ \sin^2\theta$

$= \sin\theta(1-\cos^2\theta)$

$= \sin\theta - \sin\theta.\cos^2\theta$

L.H.S = R.H.S

Q.22. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

Solution: Let

L.H.S $= \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= (1)(\cos^2\theta - \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta$

L.H.S = R.H.S

Q.23. $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \dfrac{\sin\theta}{1-\cos\theta}$

Solution: Let

$$\text{L.H.S} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2-(\cos\theta)^2}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}\times\frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{(1)^2-(\cos\theta)^2}{\sin\theta(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin\theta}{1-\cos\theta}$$

L.H.S = R.H.S

Q.24. $\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}} = \dfrac{\sec\theta+1}{\tan\theta}$

Solution: Let

$$\text{L.H.S} = \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$$

$$= \sqrt{\frac{\sec\theta+1}{\sec\theta-1}\times\frac{\sec\theta+1}{\sec\theta+1}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{(\sec\theta)^2-(1)^2}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}}$$

$$= \frac{\sec\theta+1}{\tan\theta}$$

L.H.S = R.H.S