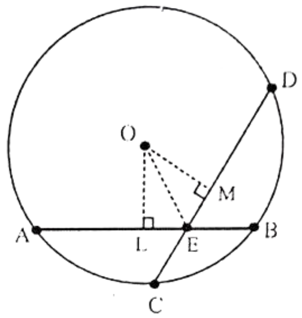
EXERCISE 9.2

Q.1 Two equal chords of a circle intersect, show that the segments of the one are equal corresponding to the segments of the other.

Given: A circle with centre "O". Two equal chords $\overline{AB}$ and $\overline{CD}$ (i.e. $m\overline{AB} = m\overline{CD}$) intersect each other at point E.



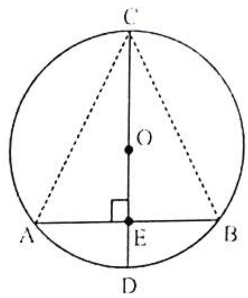
To prove: $m\overline{AE} = m\overline{ED}$ and $m\overline{EB} = m\overline{EC}$

Construction: Draw perpendiculars $\overline{OL}$ and $\overline{OM}$ from the centre "O" to the chords $\overline{AB}$ and $\overline{CD}$ respectively. L and M are midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.

Proof:

Statements	Reasons
In $\Delta OLE \leftrightarrow \Delta OME$	
$\overline{OL} \cong \overline{OM}$	Two equal chords of a circle are equidistant from the centre.
$m\angle OLE = m\angle OME = 90^\circ$	$\overline{OL} \perp \overline{AB}$ and $\overline{OM} \perp \overline{CD}$
$m\overline{OE} \cong m\overline{OE}$	Common side
$\therefore$ $\Delta OLE \cong \Delta OME$	H.S ≅ H.S
$\overline{LE} \cong \overline{ME}$ .... (i)	Corresponding sides of congruent triangles.
$m\overline{AL} = \frac{1}{2} m\overline{AB}$	
$m\overline{DM} = \frac{1}{2} m\overline{CD}$	
$m\overline{AL} = m\overline{DM}$ .... (ii)	Both are half of equal chords.
$m\overline{AL} + m\overline{LE} = m\overline{DM} + m\overline{ME}$	Adding (i) and (ii).
$m\overline{AE} = m\overline{DE}$ .... (iii)	
Now, $m\overline{AB} = m\overline{CD}$	Given
$m\overline{AE} + m\overline{EB} = m\overline{DE} + m\overline{EC}$	
$m\cancel{\overline{AE}} + m\overline{EB} = m\cancel{\overline{AE}} + m\overline{EC}$	From (iii)
$m\overline{EB} = m\overline{EC}$	By cancellation property.

Q.2 **AB is the chord of a circle and diameter CD is perpendicular bisector of $\overline{AB}$. Prove that $m\overline{AC} = m\overline{BC}$**



Given: A circle with centre "O" diameter $\overline{CD} \perp$ chord $\overline{AB}$ i.e

$m\angle CEA = m\angle CEB = 90°$ and $\overline{AE} \cong \overline{EB}$

To prove: $m\overline{AC} = m\overline{BC}$

Construction: Join C to A and B.

Proof:

Statements	Reasons
In $\Delta ACE \leftrightarrow \Delta BCE$	
$\overline{AE} \cong \overline{EB}$	A diameter $\overline{CD} \perp$ on chord AB bisect it.
$m\angle CEA = m\angle CEB$	Given
$\overline{CE} \cong \overline{CE}$	Common side
$\Delta ACE \cong \Delta BCE$	S.A.S ≅ S.A.S
$\overline{AC} \cong \overline{BC}$	Corresponding sides of congruent triangles.
$m\overline{AC} = m\overline{BC}$	

Q.3 **As shown in fig. find the distance between two parallel chords $\overline{AB}$ and $\overline{CD}$.**

Given: A fig. as shown.

To find: Distance between two parallel chords $\overline{AB}$ and $\overline{CD}$ i.e $m\overline{EF} = ?$

Construction: Join O to A.

Calculations:

$$m\overline{AE} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(6cm) = 3cm$$

$$m\overline{CF} = \frac{1}{2} m\overline{CD} = \frac{1}{2}(8cm) = 4cm$$

$$m\overline{OA} = m\overline{OC} = 5cm$$

In right triangle AOE, by Pythagoras theorem

$$\left(m\overline{OA}\right)^2 = \left(m\overline{OE}\right)^2 + \left(m\overline{AE}\right)^2$$

$$(5cm)^2 = \left(m\overline{OE}\right)^2 + (3cm)^2$$

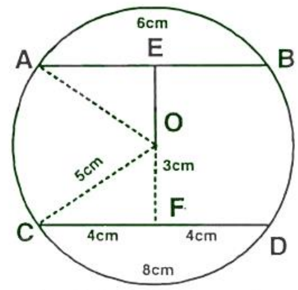
$$25cm^2 = \left(m\overline{OE}\right)^2 + (9cm^2)$$

$$m25cm^2 - 9cm^2 = \left(m\overline{OE}\right)^2$$

$$16cm^2 = \left(m\overline{OE}\right)^2$$

$$\sqrt{\left(m\overline{OE}\right)^2} = \sqrt{16cm^2}$$

$$m\overline{OE} = 4cm$$



In right triangle COF, by Pythagoras theorem

$$\left(m\overline{OC}\right)^2 = \left(m\overline{CF}\right)^2 + \left(m\overline{OF}\right)2$$

$$(5cm)^2 = (4cm)^2 + \left(m\overline{OF}\right)^2$$

$$25cm^2 - 16cm^2 + \left(m\overline{OF}\right)^2$$

$$9cm^2 = \left(m\overline{OF}\right)^2$$

$$\sqrt{\left(m\overline{OF}\right)^2} = \sqrt{9cm^2}$$

$$\boxed{m\overline{OF} = 3cm}$$

Now,

$$m\overline{EF} = m\overline{OE} + m\overline{OF}$$

$$m\overline{EF} = 4cm + 3cm$$

$$\boxed{m\overline{EF} = 7cm}$$