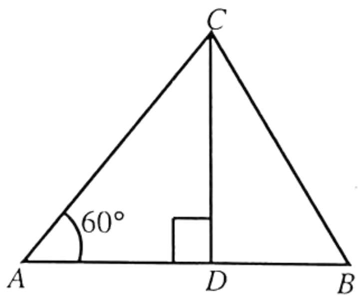
MISCELLANEOUS EXERCISE – 8

Q. 1 In a ΔABC, m∠A = 60°,

Prove that $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - (m\overline{AB})(m\overline{AC})$



Solution:

Given: In a ABC, m∠A = 60°

To Prove: $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - (m\overline{AB})(m\overline{AC})$

Proof: In acute angled triangle ABC, by Theorem No. 2

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD}) \text{.......(i)}$$

In right angled Δ ACD

$$\text{Cos}60^\circ = \frac{m\overline{AD}}{m\overline{AC}} \qquad \frac{1}{2} = \frac{m\overline{AD}}{m\overline{AC}} \qquad (\ \text{Cos}60^\circ = \frac{1}{2})$$

$$m\overline{AD} = \frac{1}{2}m\overline{AC}$$

Put it in equation (i)

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})\ \frac{1}{2}\ m\overline{AC}$$

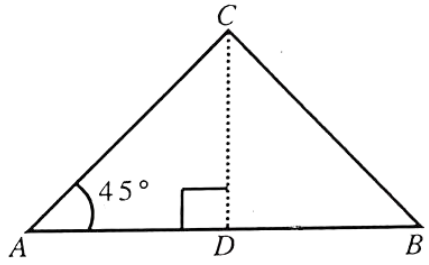
$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - \cancel{2}(m\overline{AB})\ \frac{1}{\cancel{2}}\ m\overline{AC}$$

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - (m\overline{AB})(m\overline{AC}) \qquad \text{Hence proved}$$

Q. 2 In a ΔABC, m∠A = 45°, prove that $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - \sqrt{2}(m\overline{AB})(m\overline{AC})$.

Solution:

Given: In a ABC, m∠A = 45°



To prove: $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - \sqrt{2}(m\overline{AB})(m\overline{AC})$

Proof: In triangle ABC, ∠A is acute so by Theorem 2

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD}) \text{....... (i)}$$

In right angled Δ ACD

$$\text{Cos}45^\circ = \frac{m\overline{AD}}{m\overline{AC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{AD}}{m\overline{AC}}$$

$$m\overline{AD} = \frac{1}{\sqrt{2}}m\overline{AC}$$

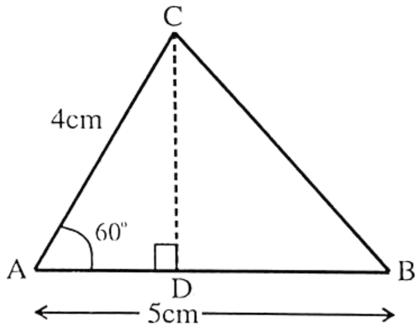
Put it in equation (i)

$$\left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\ \frac{1}{\sqrt{2}} m\overline{AC}$$

$$\left(m\overline{BC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 - \sqrt{2}\left(m\overline{AB}\right)\left(m\overline{AC}\right) \qquad \frac{2}{\sqrt{2}} = \sqrt{2}$$

Q. 3 In a ΔABC, calculate $m\overline{BC}$ when $m\overline{AB} = 5cm, m\overline{AC} = 4cm, m\angle A = 60°$

Solution:



Given: In a ΔABC $m\overline{AB} = 5cm, m\overline{AC} = 4cm, m\angle A = 60°$

To Find: $m\overline{BC} = ?$

Calculations: In triangle ABC, $\angle A$ is acute so by Theorem 2

$$\left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$$

$$\left(m\overline{BC}\right)^2 = (4cm)^2 + (5cm)^2 - 2(5cm)\left(m\overline{AD}\right) \text{......(i)}$$

In right angle ΔACD

$$\text{Cos}60° = \frac{m\overline{AD}}{m\overline{AC}} \qquad \frac{1}{2} = \frac{m\overline{AD}}{4cm} \qquad \left(\text{Cos}60° = \frac{1}{2}\right)$$

$$2cm = m\overline{AD} \qquad \boxed{m\overline{AD} = 2cm}$$

Putting the corresponding values in equation (i)

$$\left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$$

$$= (4cm)^2 + (5cm)^2 - 2(5cm)(2cm)$$

$$= 16cm^2 + 25cm^2 - 20cm^2$$

$$= 41cm^2 - 20cm^2$$

$$\left(m\overline{BC}\right)^2 = 21cm^2$$

$$\sqrt{\left(m\overline{BC}\right)^2} = \sqrt{21cm^2} \qquad \boxed{m\overline{BC} = 4.58\ cm}$$

Q. 4 In a ΔABC, calculate $m\overline{AC}$ when $m\overline{AB} = 5cm, m\overline{BC} = 4\sqrt{2}cm, m\angle B = 45°$

Solution:

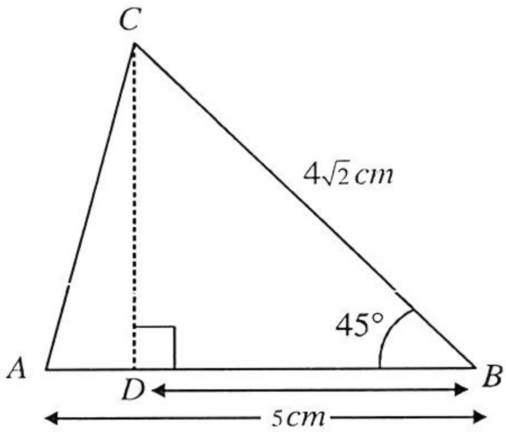
Given: In a ΔABC $m\overline{AB} = 5cm, m\overline{BC} = 4\sqrt{2}cm, m\angle B = 45°$

To Find: $m\overline{AC} = ?$

Calculations: In acute angled triangle ABC by theorem 2

$$\left(m\overline{AC}\right)^2=\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2-2\left(m\overline{AB}\right)\left(m\overline{BD}\right)$$

$$\left(m\overline{AC}\right)^2=(5cm)^2+\left(4\sqrt{2}cm\right)^2-2(5cm)\left(m\overline{BD}\right)\ ....(i)$$



$m\overline{BD}$ = ?

In right angle ΔBCD

$$\text{Cos}45° = \frac{m\overline{BD}}{m\overline{BC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{BD}}{4\sqrt{2}}$$

$$1 = \frac{m\overline{BD}}{4}$$

$$\boxed{m\overline{BD} = 4cm}$$

Putting the value of $m\overline{BD}$ in equation (i)

$$\left(m\overline{AC}\right)^2 = (5cm)^2+\left(4\sqrt{2}cm\right)^2-2(5cm)(4cm)$$

$$= 25cm^2+16(2cm^2)-40cm^2$$

$$= 25cm^2+32cm^2-40cm^2$$

$$= 57cm^2-40cm^2$$

$$\left(m\overline{AC}\right)^2 = 17cm^2$$

$$\sqrt{\left(m\overline{AC}\right)^2} = \sqrt{17cm^2} \qquad \boxed{m\overline{AC} = 4.12\text{ cm}}$$

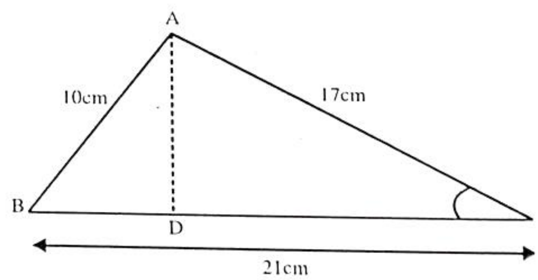
Q. 5 In a triangle ABC, $m\overline{BC}=21cm$, $m\overline{AC}=17cm$, $m\overline{AB}=10cm$. Measure the length of projection of $\overline{AC}$ upon $\overline{BC}$.

Solution:

Given: In a triangle ABC, $m\overline{BC}=21cm$, $m\overline{AC}=17cm$, $m\overline{AB}=10cm$

To Find: Projection of $\overline{AC}$ upon $\overline{BC}$ i.e., $m\overline{DC}$ = ?

Calculations: In triangle ABC, $\angle C$ is acute so by theorem 2



$$\left(m\overline{AB}\right)^2=\left(m\overline{AC}\right)^2+\left(m\overline{BC}\right)^2-2\left(m\overline{BC}\right)\left(m\overline{DC}\right)$$

$$(10cm)^2=(17cm)^2+(21cm)^2-2(21cm)\left(m\overline{DC}\right)$$

$$100cm^2=289cm^2+441cm^2-42cm\left(m\overline{DC}\right)$$

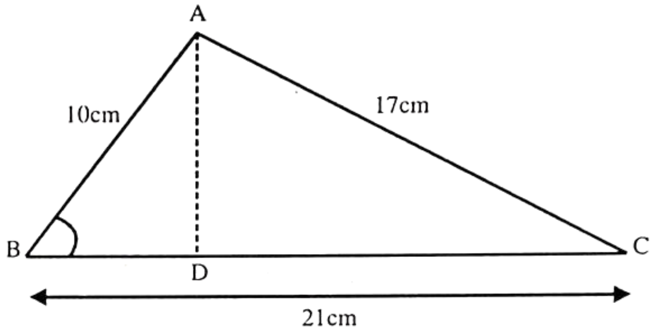
$$100cm^2-289cm^2-441cm^2=-42cm\left(m\overline{DC}\right)$$

$$-630cm^2=-42cm\left(m\overline{DC}\right)$$

$$\frac{-630cm^2}{-42cm}=m\overline{DC} \qquad \boxed{m\overline{DC}=15cm}$$

Q. 6 **In a triangle ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$, $m\overline{AB} = 10cm$. Calculate the projection of $\overline{AB}$ upon $\overline{BC}$.**

Solution:



Given

$m\overline{BC} = 21cm$

$m\overline{AC} = 17cm$

$m\overline{AB} = 10cm$

To Find: Projection of $\overline{AB}$ upon $\overline{BC}$ i.e $m\overline{BD} = ?$

Calculations:

In triangle ABC, $\angle B$ is acute so by theorem 2

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 - 2\left(m\overline{BC}\right)\left(m\overline{BD}\right)$$

$$(17cm)^2 = (10cm)^2 + (21cm)^2 - 2(21cm)\left(m\overline{BD}\right)$$

$$289cm^2 = 100cm^2 + 441cm^2 - 42cm\left(m\overline{BD}\right)$$

$$289cm^2 = 541cm^2 - 42cm\left(m\overline{BD}\right)$$

$$289cm^2 - 541cm^2 = -42cm\left(m\overline{BD}\right)$$

$$-252cm^2 = -42cm\left(m\overline{BD}\right)$$

$$\frac{-252cm^2}{-42cm} = m\overline{BD}$$

$$\boxed{m\overline{BD} = 6cm}$$

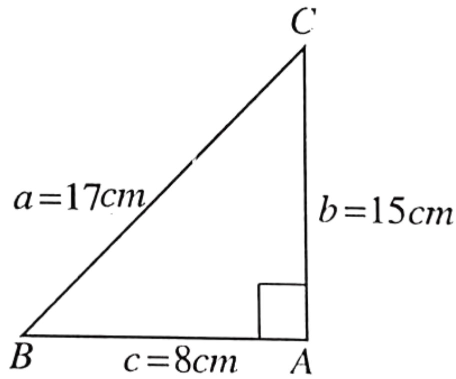
Q. 7 **In a ΔABC, $a = 17cm$, $b = 15cm$ and $c = 8cm$. Find $m\angle A$.**

Solution:

Given: In a ΔABC, $a = 17cm$, $b = 15cm$, $c = 8cm$

To Find: $m\angle A = ?$

Calculations:



Sum of squares of two sides $= b^2 + c^2$

$= (15cm)^2 + (8cm)^2$

$= 225cm^2 + 64\ cm^2$

$= 289\ cm^2$ ….. (i)

Square of length of third side $= a^2$

$= (17cm)^2$

$= 289cm^2$ ….. (ii)

From (i) and (ii)

$$a^2 = b^2 + c^2$$

The result show that the triangle ABC is right angled triangle with side a = 17cm as hypotenuse.

The angle opposite to the hypotenuse is right angle i.e $m\angle A = 90°$

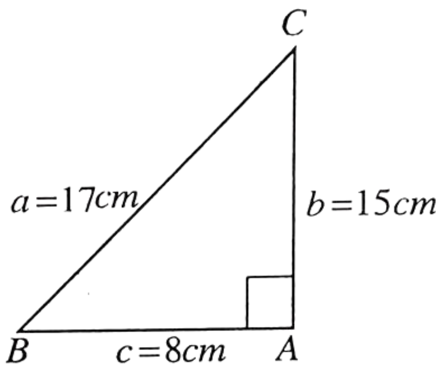
8. In a ΔABC, $a = 17cm$, $b = 15cm$ and $c = 8cm$ find $m\angle B$.

Solution:

Given: In a ΔABC, $a = 17cm$, $b = 15cm$, $c = 8cm$

To Find: $m\angle B = ?$

Calculations:



Sum of squares of two sides $= b^2 + c^2$

$= (15cm)^2 + (8cm)^2$

$= 225cm^2 + 64\ cm^2$

$= 289\ cm^2$ ..... (i)

Square of length of third side $= a^2$

$= (17cm)^2$

$= 289cm^2$ ..... (ii)

From (i) and (ii)

$$a^2 = b^2 + c^2$$

The result show that the triangle ABC is right angled triangle with $m\angle A = 90°$

In triangle ABC, $\tan m\angle B = \dfrac{Per}{Base} = \dfrac{15cm}{8cm}$

$m\angle B = \tan^{-1}\dfrac{15}{8}$ $\qquad m\angle B = (61.9)^\circ$

Q.9 Whether the triangle with sides 5cm, 7cm, 8cm is acute, obtuse or right angled.

Solution:

In a triangle ABC, let $a = 5cm$, $b = 7cm$, $c = 8cm$

Sum of squares of two sides $= a^2 + b^2$

$= (5cm)^2 + (7cm)^2$

$= 25cm^2 + 49cm^2$

$= 74cm^2$ .... (i)

Square of length of 3rd side $= c^2$

$= (8cm)^2$

$= 64cm^2$ .... (ii)

From (i) and (ii) $\quad 74cm^2 > 64cm^2$ i.e.

$$a^2 + b^2 > c^2$$

The result shows that the triangle with sides 5cm, 7cm, 8cm is acute angled triangle.
It is acute angled triangle.

Q.10 Whether the triangle with sides 8cm, 15cm, 17cm is acute, obtuse or right angled.

Solution:

In a triangle ABC let $a = 8cm$, $b = 15cm$, $c = 17cm$

Sum of squares of two sides $= a^2 + b^2$

$= (8cm)^2 + (15cm)^2$

$= 64cm^2 + 225\ cm^2$

$= 289\ cm^2$ ....(i)

Square of length of 3rd side $= c^2$

$= (17cm)^2$

$= 289cm^2$ ....(ii)

From (i) and (ii)

i.e $\quad a^2 + b^2 = c^2$

Result shows that triangle with sides $a = 8cm$, $b = 15cm$ and $c = 17cm$ is right angled triangle.