MISCELLANEOUS EXERCISE – 4

Q. 1 Multiple Choice Questions:

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. $(x+3)^2 = x^2 + 6x + 9$ is
   (a) a linear equation
   (b) an equation
   (c) an identity
   (d) none of these

2. $\dfrac{2x+1}{(x+1)(x-1)}$ is
   (a) an improper fraction
   (b) an equation
   (c) a proper fraction
   (d) none of these

3. $\dfrac{x^3+1}{(x-1)(x+2)}$ is
   (a) a proper fraction
   (b) an improper fraction
   (c) an identity
   (d) a constant term

4. **A fraction in which the degree of numerator is less than the degree of the denominator is called**
   (a) an equation
   (b) an improper fraction
   (c) an identity
   (d) a proper fraction

5. **A function of the form** $f(x) = \dfrac{N(x)}{D(x)}$, **with** $D(x) \neq 0$, **where** $N(x)$ **and** $D(x)$ **are polynomials in x is called**
   (a) an identity
   (b) an equation
   (c) a fraction
   (d) none of these

6. **The identity** $(5x+4)^2 = 25x^2 + 40x + 16$ **is true for**
   (a) one value of x
   (b) two values of x
   (c) all values of x
   (d) none of these

7. **A fraction in which the degree of the numerator is greater or equal to the degree of denominator is called**
   (a) a proper fraction
   (b) an improper fraction
   (c) an equation
   (d) algebraic relation

8. **Partial fractions of** $\dfrac{x-2}{(x-1)(x+2)}$ **are of the form**
   (a) $\dfrac{A}{x-1}+\dfrac{B}{x+2}$ (b) $\dfrac{Ax}{x-1}+\dfrac{B}{x+2}$
   (c) $\dfrac{A}{x-1}+\dfrac{Bx+C}{x+2}$ (d) $\dfrac{Ax+B}{x-1}+\dfrac{C}{x+2}$

9. **Partial fractions of** $\dfrac{x+2}{(x+1)(x^2+2)}$ **are of the form**
   (a) $\dfrac{A}{x+1}+\dfrac{B}{x^2+2}$
   (b) $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+2}$
   (c) $\dfrac{Ax+B}{x+1}+\dfrac{C}{x^2+2}$
   (d) $\dfrac{A}{x+1}+\dfrac{Bx}{x^2+2}$

10. **Partial fractions of** $\dfrac{x^2+1}{(x+1)(x-1)}$ **are of the form**
   (a) $\dfrac{A}{x+1}+\dfrac{B}{x-1}$
   (b) $1+\dfrac{A}{x+1}+\dfrac{Bx+C}{x-1}$
   (c) $1+\dfrac{A}{x+1}+\dfrac{B}{x-1}$
   (d) $\dfrac{Ax+B}{(x+1)}+\dfrac{C}{x-1}$

ANSWER KEY

1.	c	**2.**	c	**3.**	b	**4.**	d	**5.**	c
6.	c	**7.**	b	**8.**	a	**9.**	b	**10.**	c

Q. 2 Write short answers of the following questions:

(i) Define a rational fraction.

An expression of the form $\frac{N(x)}{D(x)}$ with $D(x) \neq 0$ and N(x) and D(x) are polynomials in x with real coefficients, is called a rational fraction. Every fractional expression can be expressed as a quotient of two polynomials.

(ii) What is a proper fraction?

A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called a proper fraction if degree of the polynomial N(x) in the numerator is less than the degree of the polynomial D(x) in the denominator.

(iii) What is an improper fraction?

A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction if degree of the polynomial N(x) is greater or equal to the degree of the polynomial D(x) e.g $\frac{x^2+1}{x-1}$

(iv) What are partial fractions?

Every proper fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ can be resolved into an algebraic sum of components fractions. These components fractions of a resultant fraction are called its partial fractions.

(v) How can we make partial fractions of $\frac{x-2}{(x+2)(x+3)}$?

Solution: $\frac{x-2}{(x+2)(x+3)}$

Let $\frac{x-2}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$ ......(i)

Multiplying both sides by (x+2)(x+3), we get

$x-2 = A(x+3) + B(x+2)$ .... (ii)

As both sides of the identity are equal for all values of x,

Put $x+2=0$ i.e $x=-2$ in equation (ii), we get

$-2-2 = A(-2+3)$

$-4 = A$

$\Rightarrow \boxed{A = -4}$

Now put $x+3=0$ i.e $x=-3$ in equation (ii) we get

$-3-2 = B(-3+2)$

$-5 = -B$

$\Rightarrow \boxed{B = 5}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x-2}{(x+2)(x+3)} = \frac{-4}{x+2} + \frac{5}{x+3}$$

(vi) Resolve $\frac{1}{x^2-1}$ into partial fractions.

Solution: $\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$

Let $\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$ .... (i)

Multiplying both sides By $(x-1)(x+1)$, we get

$1 = A(x+1) + B(x-1)$ ....(ii)

As both sides of identity are equal for all values of x

Putting $x-1=0$ i.e $x=1$ in equation (ii) we get

$1 = A(1+1)$

$1 = 2A$

$\Rightarrow \boxed{A = \frac{1}{2}}$

Putting $x+1=0$ i.e $x=-1$ in equation (ii) we get

$1 = B(-1-1)$

$1 = -2B$

$\Rightarrow \boxed{B = -\frac{1}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{1}{x^2-1} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$$

(vii) **Find partial fractions of** $\frac{3}{(x+1)(x-1)}$

Solution: $\frac{3}{(x+1)(x-1)}$

Let $\frac{3}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ ....(i)

Multiplying both sides by (x+1) (x − 1), we get

$3 = A(x-1) + B(x+1)$ ....(ii)

As both sides of the identity are equal for all values of x.

Put x + 1= 0 i.e x = −1put in equation (ii) we get

$3 = A(-1-1)$

$3 = -2A \Rightarrow \boxed{A = \frac{-3}{2}}$

Now put $x - 1 = 0$ i.e x = 1 in equation (ii) we get

$\Rightarrow 3 = B(1+1)$

$3 = 2B \Rightarrow \boxed{B = \frac{3}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{3}{(x+1)(x-1)} = \frac{-3}{2(x+1)} + \frac{3}{2(x-1)} = \frac{3}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

(viii) Resolve $\frac{x}{(x-3)^2}$ **into partial fractions.**

Solution: $\frac{x}{(x-3)^2}$

Let $\frac{x}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2}$ ....(i)

Multiplying both sides by $(x-3)^2$, we get

$x = A(x-3) + B$ ....(ii)

As both sides of the identity are equal for all values of x,

Put x − 3 = 0 i.e x =3 in equation (ii) we get

$3 = B$

$\Rightarrow \boxed{B = 3}$

Now comparing the coefficients of x, we have

$\Rightarrow \boxed{A = 1}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x}{(x-3)^2} = \frac{1}{x-3} + \frac{3}{(x-3)^2}$$

(ix) How we can make the partial fractions of $\frac{x}{(x+a)(x-a)}$**?**

Solution: $\frac{x}{(x+a)(x-a)}$

Let $\frac{x}{(x+a)(x-a)} = \frac{A}{x+a} + \frac{B}{x-a}$ ....(i)

Multiplying both sides by (x + a) (x − a), we get

$x = A(x-a) + B(x+a)$ .... (ii)

As both sides of the identity are equal for all values of x,

Put x + a = 0 i.e x = −a put in equation (ii) we get

$-a = A(-a-a)$

$-a = -2aA$

$\Rightarrow A = \frac{-a}{-2a}$

$\Rightarrow \boxed{A = \frac{1}{2}}$

Now put x − a = 0 i.e x = a in equation (ii) we get

$a = B(a+a)$

$a = 2aB$

$\Rightarrow B = \frac{a}{2a}$

$\Rightarrow \boxed{B = \frac{1}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x}{(x+a)(x-a)} = \frac{1}{2(x+a)} + \frac{1}{2(x-a)}$$

$$= \frac{1}{2}\left(\frac{1}{x+a} + \frac{1}{x-a}\right)$$

(x) Whether $(x+3)^2 = x^2 + 6x + 9$ is an identity?

Answer:

Yes $(x+3)^2 = x^2 + 6x + 9$ is an identity because it is true for all the values of x.